Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8140996719

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/63**

Paper 6 Investigation and Modelling (Extended) **October/November 2020**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 4) and part **B** (Questions 5 to 8).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

Answer **both** parts **A** and **B**.

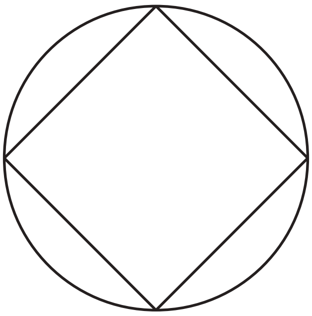
A INVESTIGATION (QUESTIONS 1 TO 4)

AREAS OF POLYGONS INSIDE AND OUTSIDE A CIRCLE (30 marks)

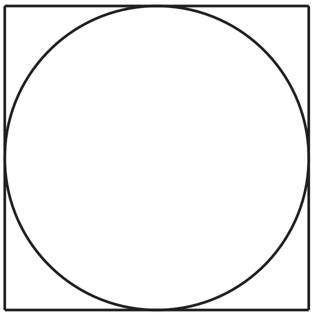
You are advised to spend no more than 50 minutes on this part.

This investigation looks at the areas of polygons drawn inside and outside a circle of radius 10 cm.

An inscribed polygon is a polygon in which all the vertices lie on a circle.
This is an inscribed square.



A circumscribed polygon is a polygon in which each side is a tangent to a circle.
This is a circumscribed square.



You may find some of these formulas useful.

Area, A, of circle, radius r $\quad A = \pi r^2$

Area, A, of triangle, base b, height h $\quad A = \frac{1}{2}bh$

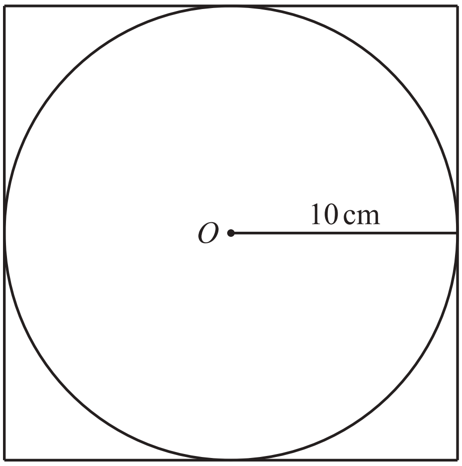
In a right-angled triangle,

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}},$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}},$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}.$$

1 **(a)**



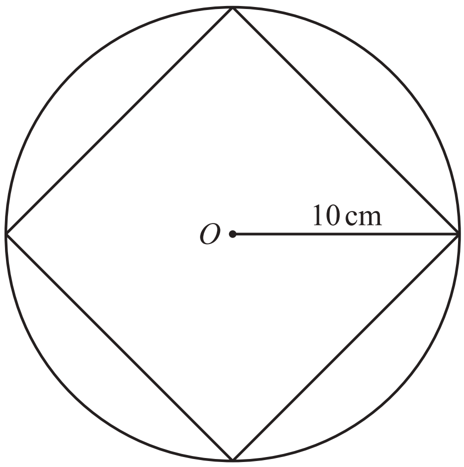
NOT TO SCALE

A square circumscribes a circle, centre O, radius 10 cm.

Work out the area of the square.

.... [1]

(b)



NOT TO SCALE

A square is inscribed in a circle, centre O, radius 10 cm.

Work out the area of the square.

.... [2]

(c) Show that the area of a circle, radius 10 cm, is $100\pi\,\text{cm}^2$.

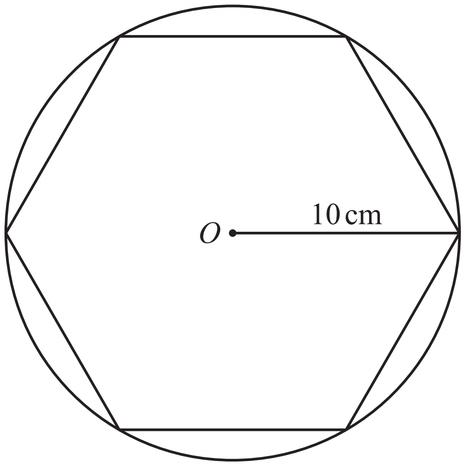
[1]

(d) Area of inscribed square < Area of circle < Area of circumscribed square

Use this statement to complete the inequality below.

.... $< \pi <$ .... [1]

2 **(a)**



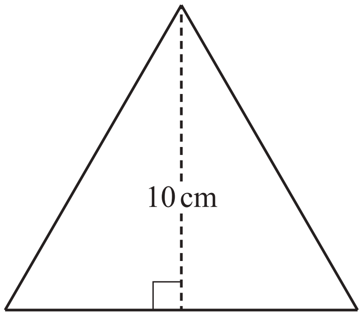
NOT TO SCALE

A regular hexagon is inscribed in a circle, centre O, radius 10 cm.

Find the area of the hexagon.

.... [3]

(b) **(i)**



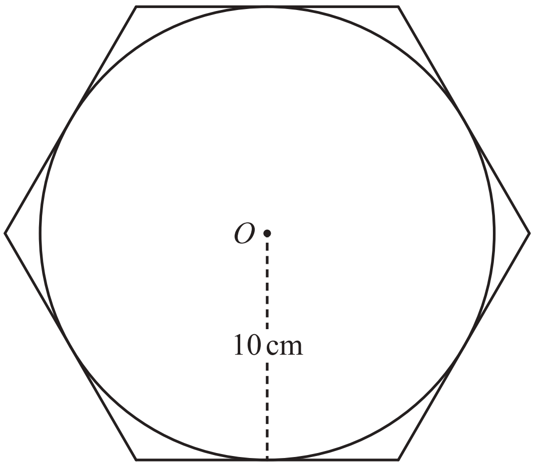
NOT TO SCALE

An equilateral triangle has height 10 cm.

Find the area of the triangle.

.... [3]

(ii)



NOT TO SCALE

A regular hexagon circumscribes a circle, centre *O*, radius 10 cm.

Using your answer to **part (i)**, find the area of the hexagon.

.... [2]

(c) **(i)** Use **Question 1(c)**, **Question 2(a)** and **Question 2(b)(ii)** to complete the inequality.

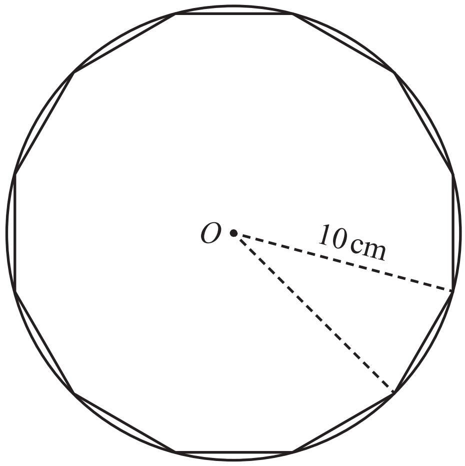
.... $<$ π $<$ .... [1]

(ii) Give a geometric reason why the range in the inequality in **Question 2(c)(i)** is smaller than the range in the inequality in **Question 1(d)**.

....

.... [1]

3 (a)



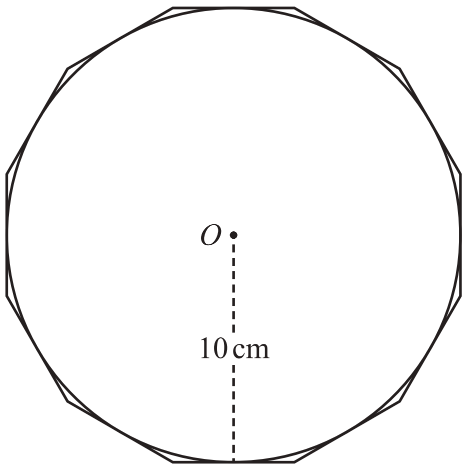
NOT TO SCALE

A regular 12-sided polygon is inscribed in a circle, centre O, radius 10 cm.

Find the area of this polygon.

.... [2]

(b)



NOT TO SCALE

A regular 12-sided polygon circumscribes a circle, centre O, radius 10 cm.

Find the area of this polygon.

.... [3]

(c) Use the answers to **part (a)** and **part (b)** to complete the inequality.

.... $<\ \pi\ <$ .... [1]

4 **(a)** **(i)** Show that a formula for the area, $A\,\text{cm}^2$, of a regular polygon with n sides **inscribed** in a circle, radius 10 cm, is

$$A = 50n\sin\left(\frac{360}{n}\right)^{\circ}.$$

[2]

(ii) Show that a formula for the area, $B\,\text{cm}^2$, of a regular polygon with n sides that **circumscribes** a circle, radius 10 cm, is

$$B = 100n\tan\left(\frac{180}{n}\right)^{\circ}.$$

[2]

(b) **(i)** Work out the area of a regular polygon with 100 sides that is **inscribed** in a circle, radius 10 cm.
Give your answer correct to 4 significant figures.

.... [2]

(ii) Work out the area of a regular polygon with 100 sides that **circumscribes** a circle, radius 10 cm.
Give your answer correct to 4 significant figures.

.... [2]

(c) Use your answers to **part (b)** to explain how you can find the value of π correct to 3 significant figures.

....

.... [1]

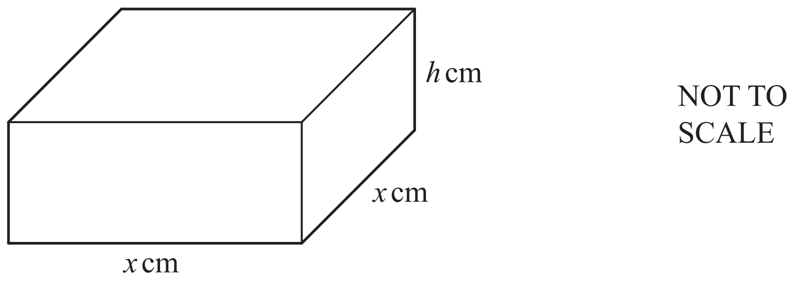
B MODELLING (QUESTIONS 5 TO 8)

MODELLING CONTAINERS (30 marks)

You are advised to spend no more than 50 minutes on this part.

Olivia wants to design a closed container with a volume of $1000\,\text{cm}^3$ and minimum surface area.

5 Olivia uses a square-based cuboid to model the container.



(a) **(i)** Write down a formula for the volume of the cuboid, $V\,\text{cm}^3$, in terms of x and h.

.... [1]

(ii) Find a formula for the surface area, $S\,\text{cm}^2$, of the cuboid, in terms of x and h.
Give your answer in its simplest form.

.... [2]

(b) **(i)** $V = 1000$.

Write h in terms of x.

.... [1]

(ii) Show that $S = 2x^2 + \dfrac{4000}{x}$.

[1]

(iii) Work out the value of S when $x = 25$.

.... [1]

(c) Sketch the graph of $S = 2x^2 + \frac{4000}{x}$ for $0 < x \leqslant 25$.

[3]

(d) **(i)** Find the minimum surface area of the cuboid.

.... [1]

(ii) Describe the container that gives the minimum surface area for Olivia's model.

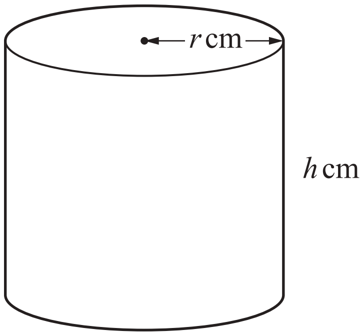
....

.... [2]

6

Volume, V, of a cylinder of radius r, height h	$V = \pi r^2 h$
Curved surface area, A, of a cylinder of radius r, height h	$A = 2\pi rh$

Olivia now uses a cylinder to model the container.



NOT TO SCALE

The total surface area of this model is $T\,\text{cm}^2$.

(a) $V = 1000$.

Show that $T = 2\pi r^2 + \dfrac{2000}{r}$.

[3]

(b) **(i)** Find the minimum surface area of the cylinder.

.... [2]

(ii) Find the dimensions of the cylinder with the minimum surface area.

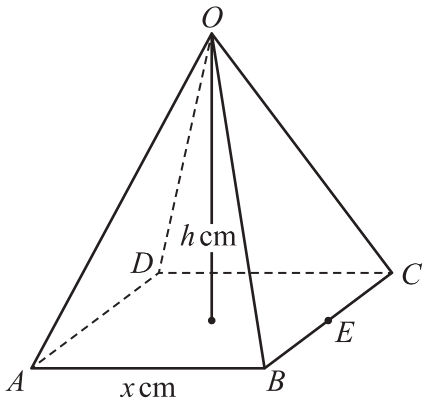
$r =$ ....

$h =$ .... [2]

7

Volume, V, of a pyramid, base area A, height h $\qquad V = \frac{1}{3}Ah$

Olivia now uses a square-based pyramid to model the container.



NOT TO SCALE

The pyramid, $OABCD$, has a square base of side x cm and height h cm.
The vertex of the pyramid, O, is directly above the centre of the square base.
E is the mid-point of BC.

(a) Find an expression for OE in terms of h and x.

.... [2]

(b) The total surface area of this model is $P\,\text{cm}^2$.

$V = 1000$.

Show that $P = x^2 + \dfrac{\sqrt{x^6 + 36\,000\,000}}{x}$.

[4]

(c) **(i)** Find the minimum surface area of the pyramid.

.... [2]

(ii) Find the dimensions of the pyramid with the minimum surface area.

$x =$ ....

$h =$ .... [2]

8 Olivia recommends the container with the smallest surface area to a company.

Give a geometric reason why the company might not accept Olivia's recommendation.

Olivia recommends the ....

Geometric reason ....

.... [1]