Cambridge Assessment International Education

# Cambridge IGCSE™

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**CAMBRIDGE INTERNATIONAL MATHEMATICS** **0607/62**

Paper 6 Investigation and Modelling (Extended) **October/November 2020**

**1 hour 40 minutes**

You must answer on the question paper.

No additional materials are needed.

**INSTRUCTIONS**

- Answer both part **A** (Questions 1 to 4) and part **B** (Questions 5 to 9).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

**INFORMATION**

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.



**[Turn over**

Answer **both** parts **A** and **B**.

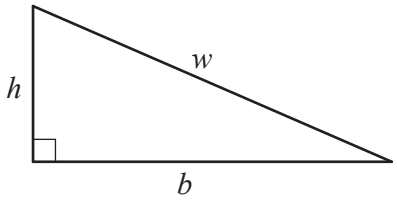
## A INVESTIGATION (QUESTIONS 1 to 4)

### AREA OF RIGHT-ANGLED TRIANGLES (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at finding the area of a right-angled triangle using its perimeter.

In this investigation all lengths are in centimetres.

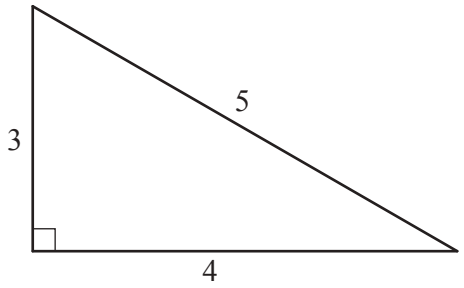



$w$ is the hypotenuse of the triangle,
$b$ is the base of the triangle,
$h$ is the height of the triangle.

Perimeter, $P$, of this triangle. $P = b + h + w$

Area, $A$, of this triangle. $A = \frac{1}{2}bh$

**1 (a)**




NOT TO SCALE

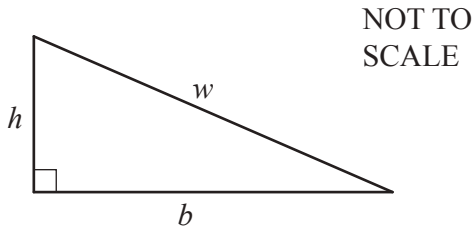
**(i)** Find the perimeter of this triangle.

..... [1]

**(ii)** Find the area of this triangle.

..... [1]

**(b)**

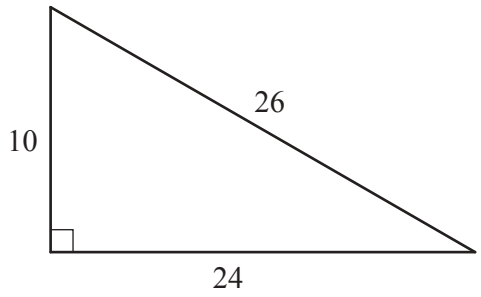



Complete the table for right-angled triangles with sides $b$, $h$ and $w$.

| $b$ | $h$ | $w$ | Perimeter, $P$ | Area, $A$ |
|---|---|---|---|---|
| 12 | 5 | 13 | 30 | 30 |
| 84 | 13 | 85 | | |
| 24 | | 25 | 56 | 84 |
| 60 | 11 | | 132 | |

[3]

**2 (a)**



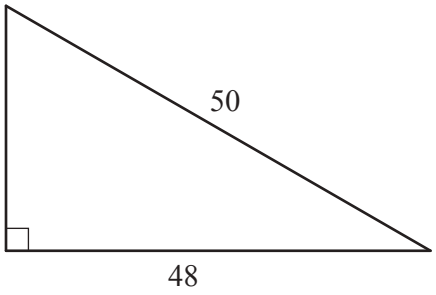

NOT TO SCALE

This triangle has perimeter $P = 60$.

Show that the calculation $\frac{60}{2} \times \left(\frac{60}{2} - 26\right)$ gives the correct area for this triangle.

[3]

**(b)**




NOT TO SCALE

This triangle has perimeter $P = 112$.

Show that the calculation $\frac{112}{2} \times \left(\frac{112}{2} - 50\right)$ gives the correct area for this triangle.

[3]

**3** **(a)** Complete the table.

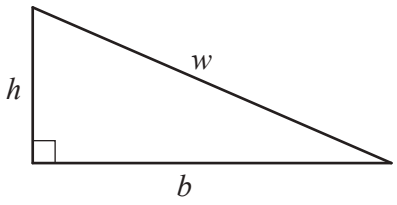
| $b$ | $h$ | $w$ | $P$ | $A$ | Calculation |
|---|---|---|---|---|---|
| 24 | 10 | 26 | 60 | 120 | $\frac{60}{2}\times\left(\frac{60}{2}-26\right) = 120$ |
| 12 | 9 | 15 | 36 | 54 | $\frac{36}{2}\times\left(\frac{36}{2}-15\right) = 54$ |
| 48 | | 50 | 112 | | $\frac{112}{2}\times\left(\frac{112}{2}-50\right) =$ |
| 15 | 8 | 17 | | 60 | $= 60$ |
| 21 | | 29 | 70 | 210 | $=$ |
| | 12 | 37 | | 210 | $=$ |

[4]

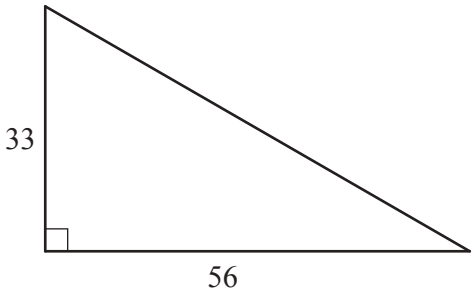
**(b)** Write an expression for the area of a right-angled triangle in terms of $P$ and $w$.

..... [1]

**(c)**




Pythagoras' Theorem
$w^2 = b^2 + h^2$

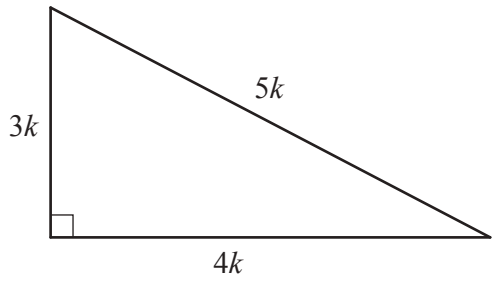



NOT TO SCALE

Use your expression from **part (b)** to find the area of this triangle.

..... [4]

**(d)**




NOT TO SCALE

Show that your expression from **part (b)** works for right-angled triangles with sides $3k$, $4k$ and $5k$.

[2]

**4** **(a)** An isosceles right-angled triangle has sides $x$, $x$ and 10.

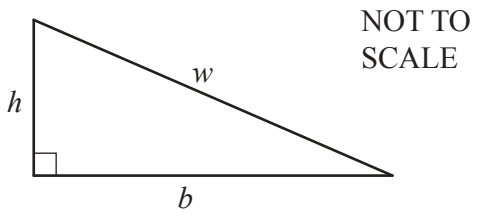
**(i)** Use **Question 3(b)** to find an expression for the area of this triangle.
Give your answer in its simplest form.

..... [2]

**(ii)** Use your answer to **part (i)** and the formula for the area of a triangle, to find the exact value of $x$.

..... [2]

**(b)**




**(i)** By writing $u = b + h$ and using your expression from **Question 3(b)**, find an expression, in terms of $u$ and $w$, for the area of any right-angled triangle.

[3]

**(ii)** Use Pythagoras' theorem to show that your expression from **part (i)** gives $\frac{1}{2}bh$ for all right-angled triangles.

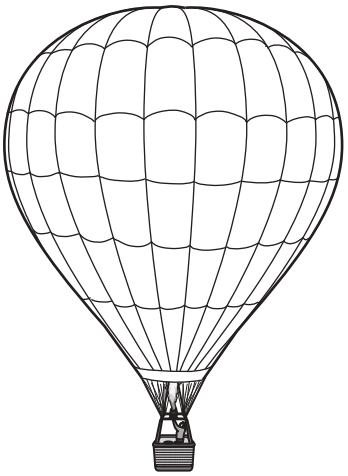
[1]

## B MODELLING (QUESTIONS 5 to 9)

### HOT AIR BALLOON FLIGHT (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task is about the flight of a hot air balloon.



A balloon travels in the direction of the wind.
The pilot can make the balloon rise or descend.

A journey is in four parts.

Part 1 Lift-off. The balloon leaves the ground and rises.

Part 2 The flight.

Part 3 The balloon descends quickly.

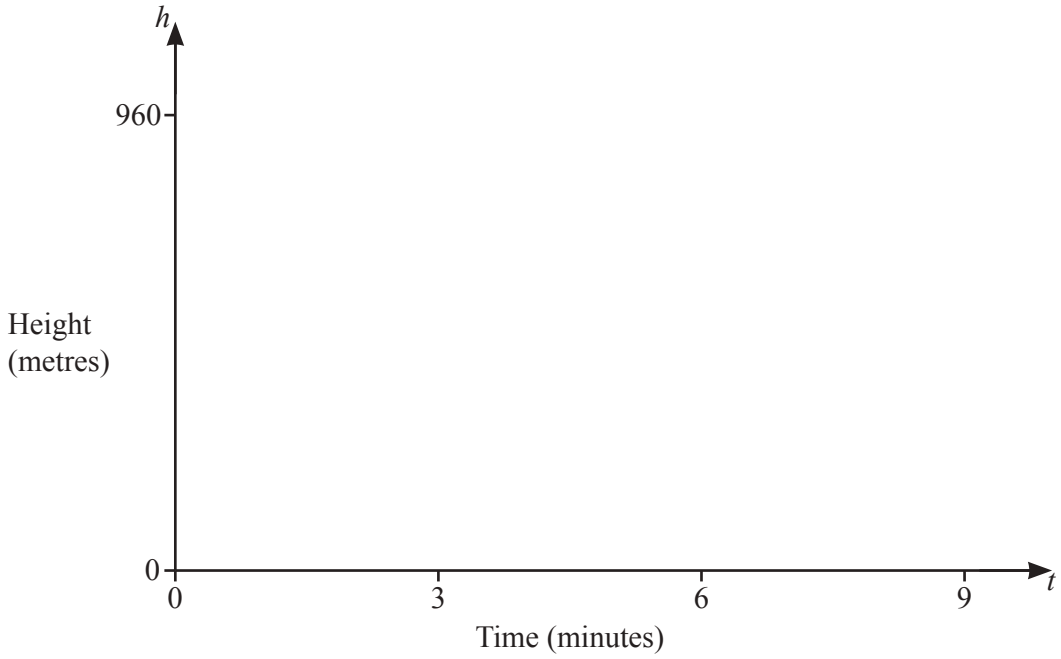
Part 4 The balloon descends slowly and lands.

**5** This journey is at sunrise.

For Part 1, a model for the height of the balloon above the ground ($h$ metres), $t$ minutes after lift-off, is

$$h = 480\left(1 - \cos(20t)°\right) \quad \text{for} \quad 0 \leqslant t \leqslant 9.$$

**(a)** On the diagram, sketch the graph of $h$ for $0 \leqslant t \leqslant 9$.




[2]

**(b)** Find the height of the balloon 3 minutes after lift-off.

..... [1]

**(c)** Find the increase in height between 3 minutes and 6 minutes after lift-off.

..... [2]

**(d)** Find the average speed at which the balloon is rising between 3 minutes and 6 minutes after lift-off. Give your answer in metres per second.

..... [3]

**(e)** Part 1 is complete 9 minutes after lift-off.

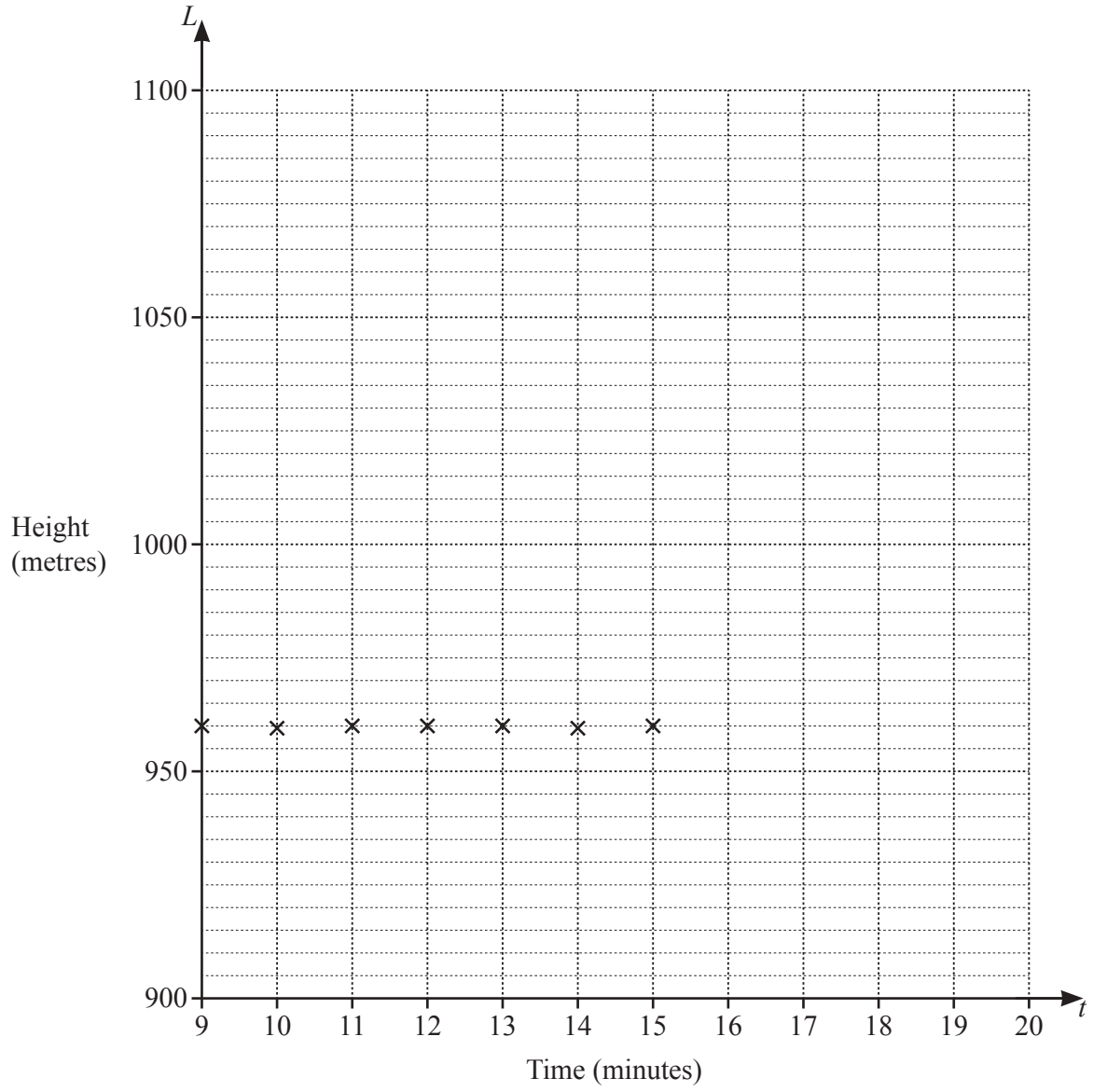
Use the model for $h$ in terms of $t$ to show that the height of the balloon at this time is 960 m.

[1]

**6** For Part 2, the table shows the height of the balloon above the ground ($L$ metres), $t$ minutes after lift-off.

| Time ($t$ minutes) | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Height ($L$ metres) | 960 | 959 | 960 | 960 | 960 | 959 | 960 | 987 | 1014 | 1041 | 1068 | 1095 |

**(a)** On the grid, complete the scatter diagram for these results.
The first seven points have been plotted for you.




[2]

**(b)** Between 15 minutes and 25 minutes after lift-off, the balloon rises at the same rate. It then travels at a constant height for 10 minutes.

Complete the list of linear functions to model $L$ for Part 2.

**(i)** For $9 < t \leqslant 15$ $L =$ .....

**(ii)** For $15 < t \leqslant 25$ $L =$ .....

**(iii)** For ..... $< t \leqslant$ ..... $L =$ .....

[5]

**7** For Part 3, the balloon descends at a constant speed of 2.5 m/s until it is 180 m above the ground.

Find how many minutes it takes the balloon to travel from **lift-off** to the end of Part 3 of the journey.

..... [4]

**8** For Part 4, a model for the height above the ground ($d$ metres), $t$ minutes after lift-off, is

$$d = \frac{450}{t - 40.125} - 60.$$

**(a)** Find how many minutes after lift-off the balloon lands.

..... [3]

**(b)** Find the average speed of the balloon during Part 4 of the journey.
Give your answer in metres per minute.

..... [2]

**Question 9 is printed on the next page.**

**9** Another journey is at sunset.

**(a)** The balloon completes Part 1 of the journey in 7.5 minutes.
At the end of Part 1, the height of the balloon above the ground is 960 m.
A model for Part 1 is $h = 480\left(1 - \cos(kt)°\right)$ for $0 \leqslant t \leqslant 7.5$ .

Find the value of $k$.

..... [2]

**(b)** In Part 2, the first 6 minutes of the journey are at a constant height of 960 m.
Then, the balloon rises 2 times as fast as in **Question 6(b)(ii)**.

Change the model in **Question 6(b)(ii)** so that it models this part of the journey.

..... [3]