Cambridge Assessment International Education

# Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*9969408613*

**CAMBRIDGE INTERNATIONAL MATHEMATICS** **0607/61**

Paper 6 Investigation and Modelling (Extended) **October/November 2020**

**1 hour 40 minutes**

You must answer on the question paper.

No additional materials are needed.

**INSTRUCTIONS**

- Answer both part **A** (Questions 1 and 2) and part **B** (Questions 3 to 6).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

**INFORMATION**

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Blank pages are indicated.

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Answer **both** parts **A** and **B**.

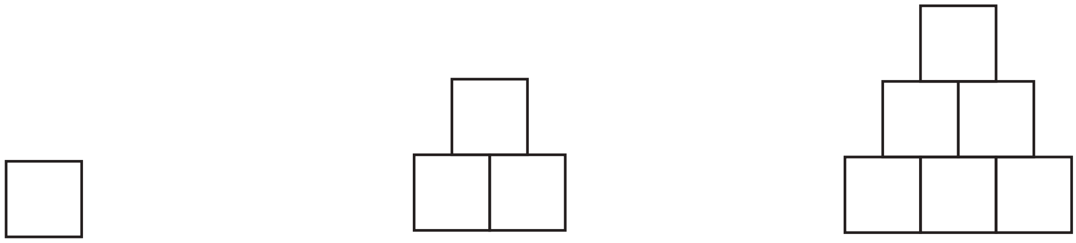
## A INVESTIGATION (QUESTIONS 1 and 2)

### PILING SQUARES (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at different ways of piling squares.
All the squares are the same size.

**1** Squares are piled in a pattern, like this:

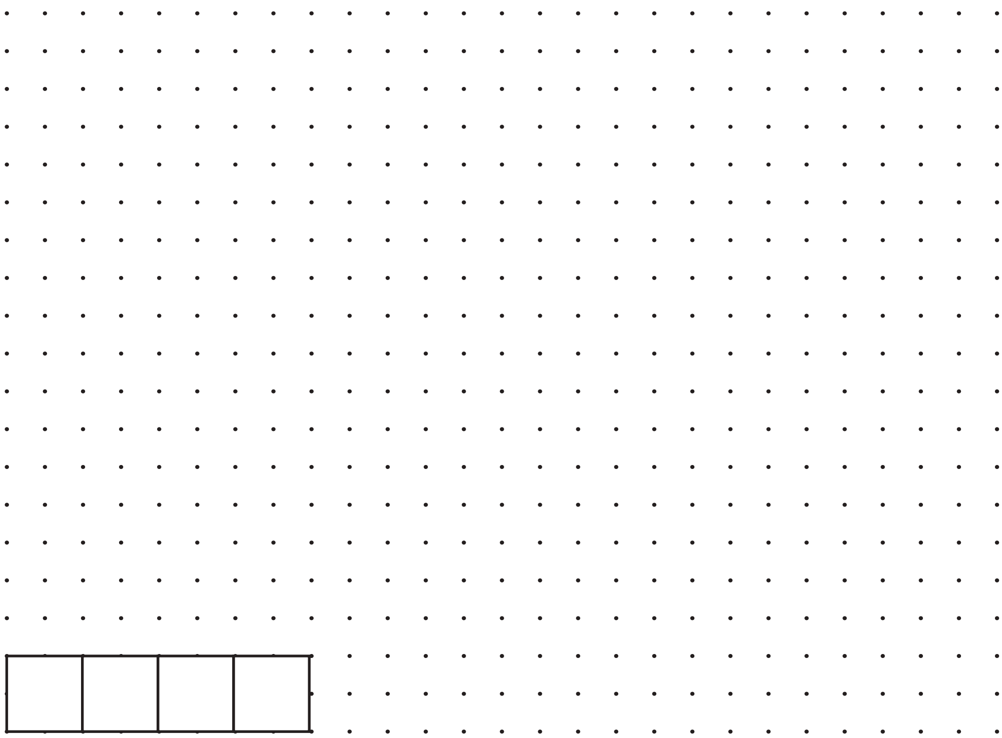


1 square on the bottom row
Total = 1 square

2 squares on the bottom row
Total = 3 squares

3 squares on the bottom row
Total = 6 squares

**(a)** On the dotty paper, complete the next diagram in the sequence.



[1]

**(b)** **(i)** Complete the table.

| Number of squares on the bottom row ($s$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Total number of squares ($T$) | 1 | 3 | 6 | | | | |

[2]

**(ii)** A formula for finding the total number of squares is $T = xs^2 + ys$.

Find the value of $x$ and the value of $y$.

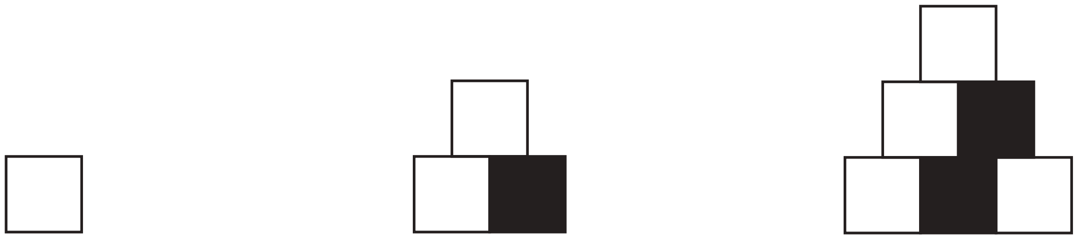
$x =$ ................................................

$y =$ ................................................ [3]

**(iii)** Show that the formula in **part (ii)** gives the correct total for a pattern of squares with 8 squares on the bottom row.

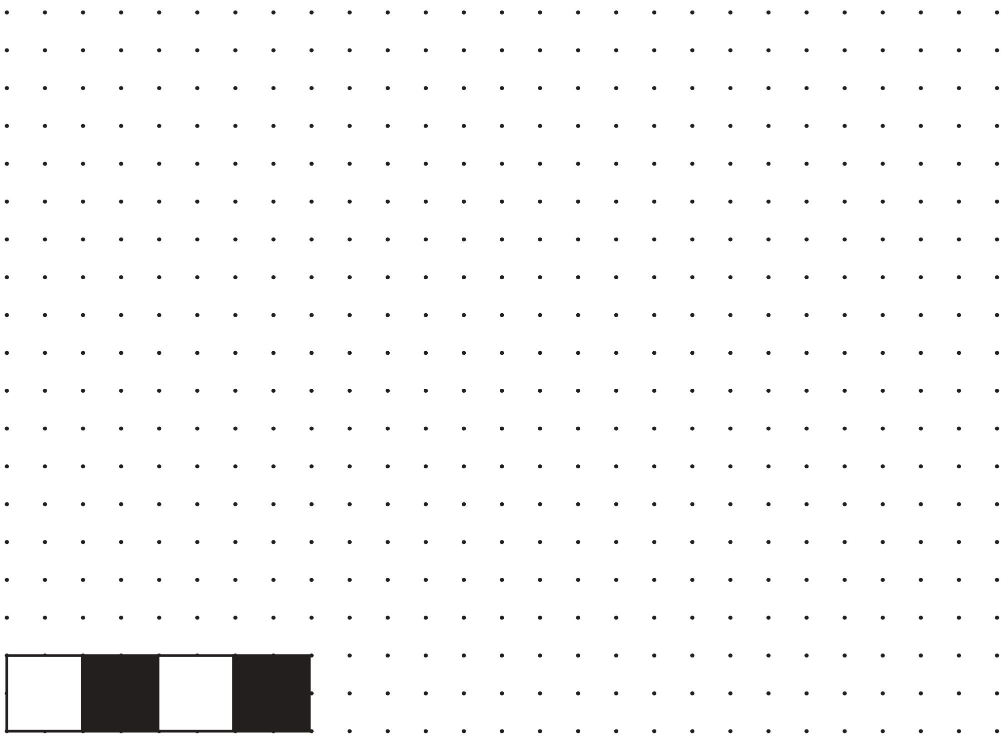
[3]

**2** Black squares and white squares are now piled on top of each other.
Every row starts with a white square, followed by a black square, followed by a white square and so on.



| 1 square on the bottom row | 2 squares on the bottom row | 3 squares on the bottom row |
|---|---|---|
| 1 white square | 2 white squares | 4 white squares |
| 0 black squares | 1 black square | 2 black squares |
| Total = 1 square | Total = 3 squares | Total = 6 squares |

**(a)** On the dotty paper, complete the next diagram in the sequence.



[1]

**(b)** Complete the table.

| Number of squares on the bottom row ($s$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Number of black squares ($b$) | 0 | 1 | 2 | | | 9 | |
| Number of white squares ($w$) | 1 | 2 | 4 | | | 12 | |
| Total number of squares ($T$) | 1 | 3 | 6 | | | 21 | |

[3]

**(c)** **(i)** Complete the two tables for black squares.
Some of the information is in the table in **part (b)**.

| Odd number of squares on the bottom row ($s$) | 1 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|
| Number of black squares ($b$) | 0 | 2 | | | 20 |

| Even number of squares on the bottom row ($s$) | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| Number of black squares ($b$) | 1 | | 9 | 16 | |

[1]

**(ii)** A formula for finding the number of black squares, $b$, when the number of squares on the bottom row, $s$, is an **odd** number is

$$b = xs^2 + y.$$

Find the value of $x$ and the value of $y$.

$x =$ ................................................

$y =$ ................................................ [3]

**(iii)** Find a formula for the number of black squares, $b$, when the number of squares on the bottom row, $s$, is an **even** number.

................................................ [2]

**(d)** **(i)** Complete the two tables for white squares.
Some of the information is in the table in **part (b)**.

| Odd number of squares on the bottom row ($s$) | 1 | 3 | 5 | 7 | 9 |
| --- | --- | --- | --- | --- | --- |
| Number of white squares ($w$) | 1 | 4 | | | 25 |

| Even number of squares on the bottom row ($s$) | 2 | 4 | 6 | 8 | 10 |
| --- | --- | --- | --- | --- | --- |
| Number of white squares ($w$) | 2 | | 12 | 20 | |

[1]

**(ii)** A formula for finding the number of white squares, $w$, when the number of squares on the bottom row, $s$, is an **odd** number is

$$w = xs^2 + ys + \frac{1}{4}.$$

Find the value of $x$ and the value of $y$.

$x =$ ................................................

$y =$ ................................................ [3]

**(iii)** Find a formula for the number of white squares, $w$, when the number of squares on the bottom row, $s$, is an **even** number.

............................................... [3]

**(e)** A pattern has a total of 253 squares.

Calculate the number of squares on the bottom row, the total number of white squares and the total number of black squares.

Number of squares on the bottom row ...............................................

Total number of white squares ...............................................

Total number of black squares ............................................... [4]

## B MODELLING (QUESTIONS 3 to 6)

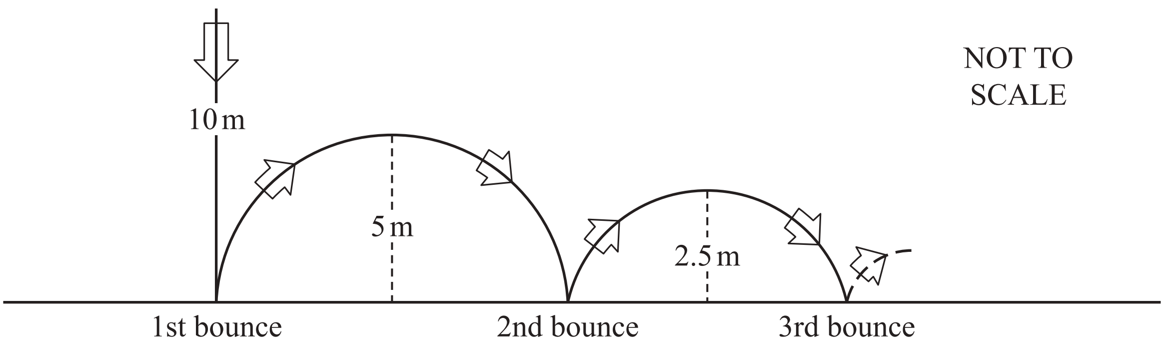
### A BOUNCING BALL (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task looks at modelling the bounce of a ball.

A ball drops vertically onto a hard floor.
Each time the ball bounces it rises vertically upwards until its speed becomes zero.
At this point the ball drops vertically to bounce on the hard floor again.

**3** A ball drops from a height of 10 metres.
Each time it bounces it rises to a maximum height which is half the height that it previously dropped.




**(a)** Calculate the maximum height of the ball after 4 bounces.

................................................ [2]

**(b)** **(i)** Calculate the first maximum height that is less than 10 cm.

................................................ [2]

**(ii)** Write down how many bounces the ball has made when its first maximum height is less than 10 cm.

................................................ [1]

**(c)** A model for the maximum height, $h$ metres, which the ball rises after $n$ bounces, is

$$h = p \times q^n .$$

For example, when $n = 1$, $h = 5$.

Find the value of $p$ and the value of $q$.

$p =$ ................................................

$q =$ ................................................ [3]

**4** A ball drops from a height of 35 metres.
After 4 bounces it reaches a maximum height of 0.056 metres.
Each time it bounces it rises to a maximum height which is a fraction of the height that it previously dropped.

Calculate the value of this fraction.

................................................ [3]

**5** A model for the total distance, $D$ metres, which a ball travels vertically, is

$$D = p\left(\frac{1+q}{1-q}\right),$$

where $p$ is the height from which the ball is dropped
and $q$ is the fraction of the previous maximum height.

**(a)** **(i)** When $q = 0$, find $D$ and explain what happens to the ball.

$D =$ ....................... and ........................................................................................

.................................................................................................................................. [1]

**(ii)** When $q = 1$, explain what happens to the ball.

..................................................................................................................................

.................................................................................................................................. [1]

**(b)** Calculate the **total** distance that the ball in **Question 3** travels vertically.

................................................. [3]

**(c)** A ball drops from a height of 40 metres.
The ball travels a total distance of 200 metres.
Each time it bounces it rises to a maximum height which is a fraction of the height that it previously dropped.

Calculate the value of this fraction.

................................................. [3]

**6** A ball drops from a height of 10 metres.
Each time it bounces it rises to a maximum height which is half the height that it previously dropped.
Time is measured in seconds.

**(a)** A model for the time taken until the $n$th bounce, $t_n$, is

$$t_n = \frac{10}{7}\left(\frac{1+\sqrt{q}-2\sqrt{q^n}}{1-\sqrt{q}}\right),$$

where $q$ is the fraction of the previous maximum height.

**(i)** Show that $t_1 = \frac{10}{7}$.

[1]

**(ii)** Show that the time taken until the 10th bounce is 8.0 seconds, correct to 2 significant figures.

[2]

**(b)** A model for $T$, the total time that the ball bounces, is

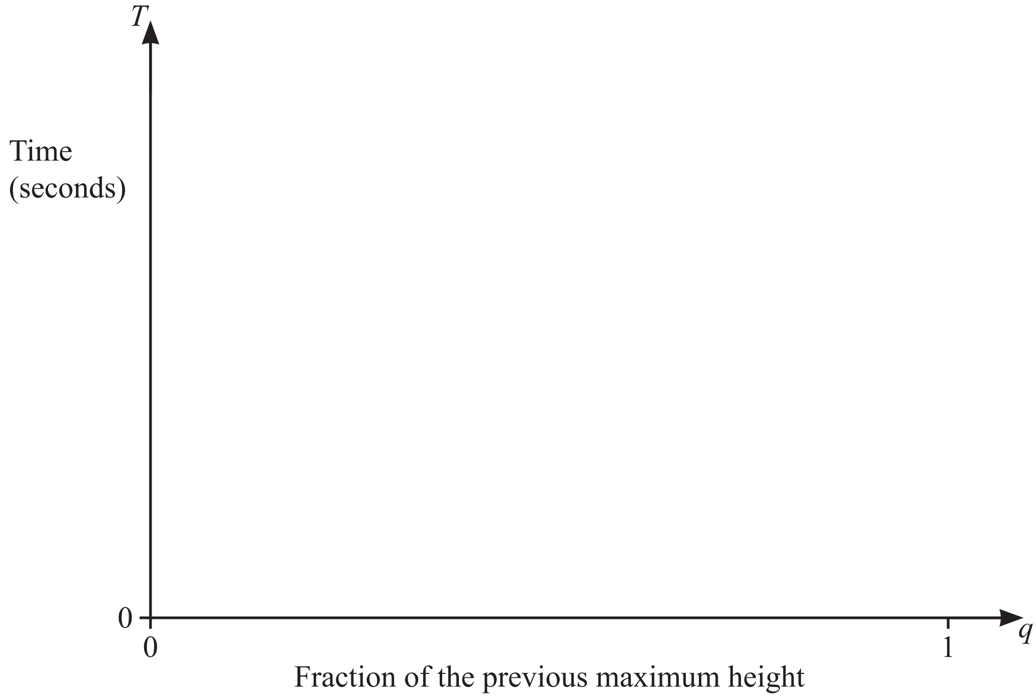
$$T = \frac{10}{7}\left(\frac{1+\sqrt{q}}{1-\sqrt{q}}\right).$$

Calculate the total time that the ball bounces.

................................................ [2]

**Questions 6(c) and 6(d) are printed on the next page.**

**(c)** Sketch the graph of the model for $T$ in **part (b)**, for $0 \leqslant q < 1$.




[4]

**(d)** The time taken until the first bounce, $t_1$, is also modelled by $t_1 = \sqrt{\frac{2p}{9.8}}$, where $p$ is the initial height that the ball drops.

**(i)** Find the value of $t_1$ when the ball is dropped from a height of 22.5 m.

................................................ [1]

**(ii)** Change the model for the total time $T$, in **part (b)**, for the ball when it is dropped from a height of 22.5 m.

................................................ [1]