Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0358135902

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/62**

Paper 6 Investigation and Modelling (Extended) **May/June 2021**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 5) and part **B** (Questions 6 to 9).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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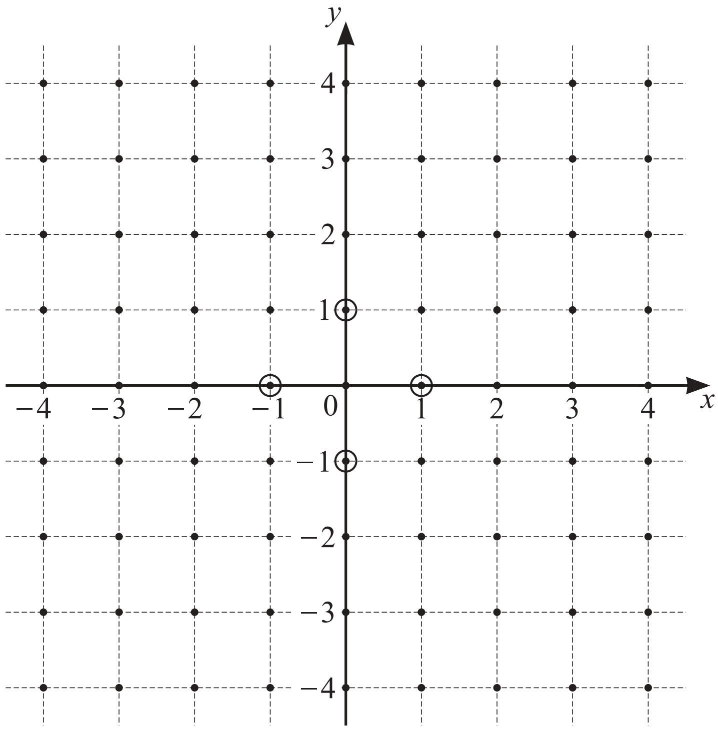
Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 TO 5)

NEAREST NEIGHBOURS (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation is about the distances between points on square and triangular grids.
All points have integer coordinates.



The point (0, 0) has 4 points closest to it.
Each point has a circle around it.
The points have coordinates (−1, 0), (0, 1), (1, 0) and (0, −1).
They are called the **1st nearest neighbours** to (0, 0).

1 The points which are next closest to (0, 0) are its **2nd nearest neighbours**.
One of these points has coordinates (1, 1).

Find the coordinates of the other 2nd nearest neighbours.

.... [2]

2 The distance from (0, 0) to its 1st nearest neighbour is 1.

Show that the distance from (0, 0) to (1, 1), a 2nd nearest neighbour, is $\sqrt{2}$.

[2]

3 The table shows information about the nth nearest neighbours to (0, 0) for values of n from 1 to 11.
d is the distance from (0, 0) to a nearest neighbour.

n	1	2	3	4	5	6	7	8	9	10	11
d^2	1	2	4	5	8	9	10	13	16	17	18
d	1	$\sqrt{2}$			$\sqrt{8}$	3	$\sqrt{10}$	$\sqrt{13}$	4	$\sqrt{17}$	$\sqrt{18}$
Number of nth nearest neighbours	4				4	4	8	8	4	8	4

(a) Complete the table. [2]

(b) Is d^2 directly proportional to n?
Show how you decide.

[2]

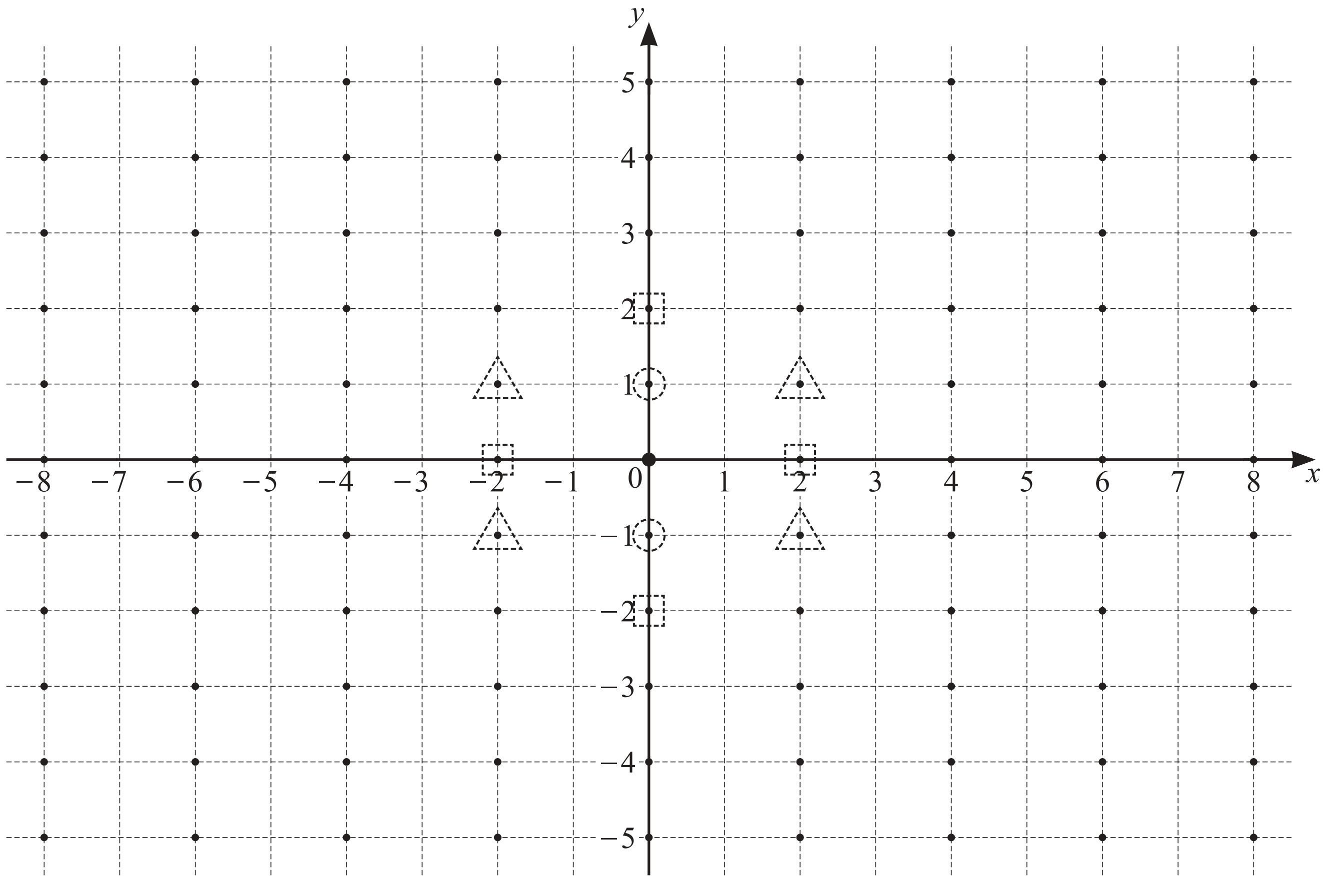
(c) The 12th nearest neighbours to (0, 0) are at a distance h, where $h^2 = 20$.

Find the coordinates of all these nearest neighbours.

....

.... [3]

4 Here is a rectangular pattern of points on a square grid.



Nearest neighbour distances can be found for these points.

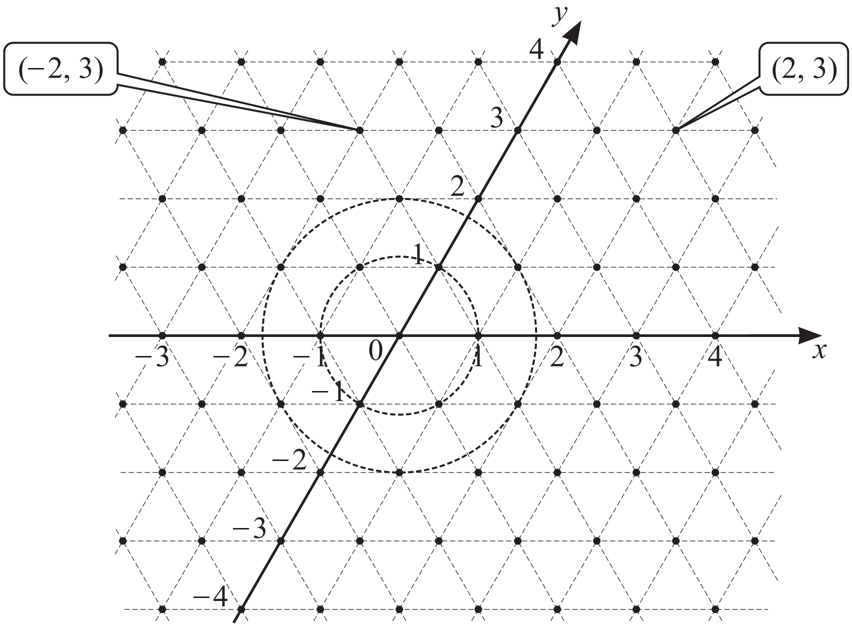
The two 1st nearest neighbours to (0, 0) have circles around them.
The four 2nd nearest neighbours to (0, 0) have squares around them.
The four 3rd nearest neighbours to (0, 0) have triangles around them.

Complete this table.

Nearest neighbour	1st	2nd	3rd	4th	5th	6th	7th
d^2	1	4	5		9	13	16
Number of nearest neighbours	2	4	4		2	4	4

[3]

5 This is a grid of points made from equilateral triangles of side 1.
These points have coordinates as shown on the grid below.



The circles show the 1st and 2nd nearest neighbours to (0, 0).
The six 1st nearest neighbours to (0, 0) have coordinates (0, 1), (1, 0), (1, −1), (0, −1), (−1, 0) and (−1, 1).

(a) Points are not always integer distances from (0, 0).

(i) Complete this table to show when a point is an integer distance from (0, 0).

Point	(1, 0)	(−1, 1)	(−1, 2)	(2, −2)	(2, 2)	(0, −3)
Integer distance	✓					
Not integer distance						

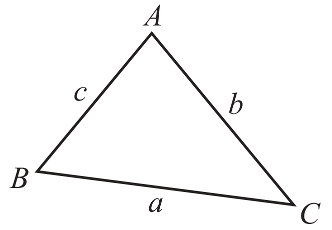
[2]

(ii) The point (a, 4) is an integer distance from (0, 0).

Write down two possible values for a.

.... [2]

(b) To calculate a distance on a triangular grid, the geometry of triangles is needed.

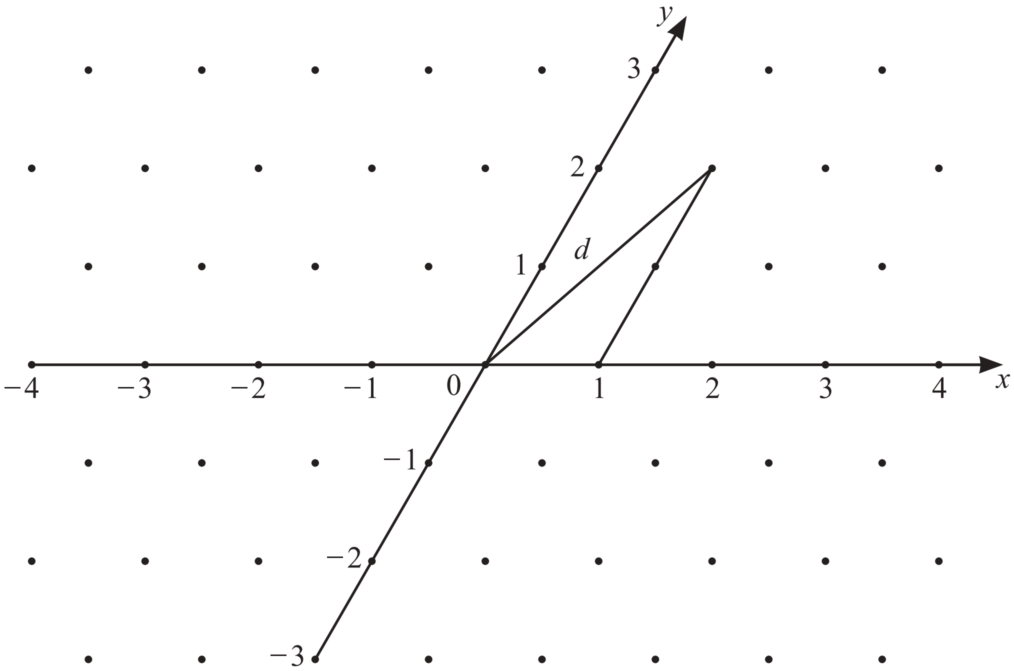


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

This grid of points is made from equilateral triangles of side length 1.

A triangle is drawn on the grid.
The triangle's vertices are at (0, 0), (1, 0) and (1, 2).
It has sides of length 1, 2 and d, the distance of (1, 2) from (0, 0).



(i) Explain why the largest angle in the triangle is 120°.

.... [1]

(ii) Calculate the distance of (1, 2) from (0, 0).

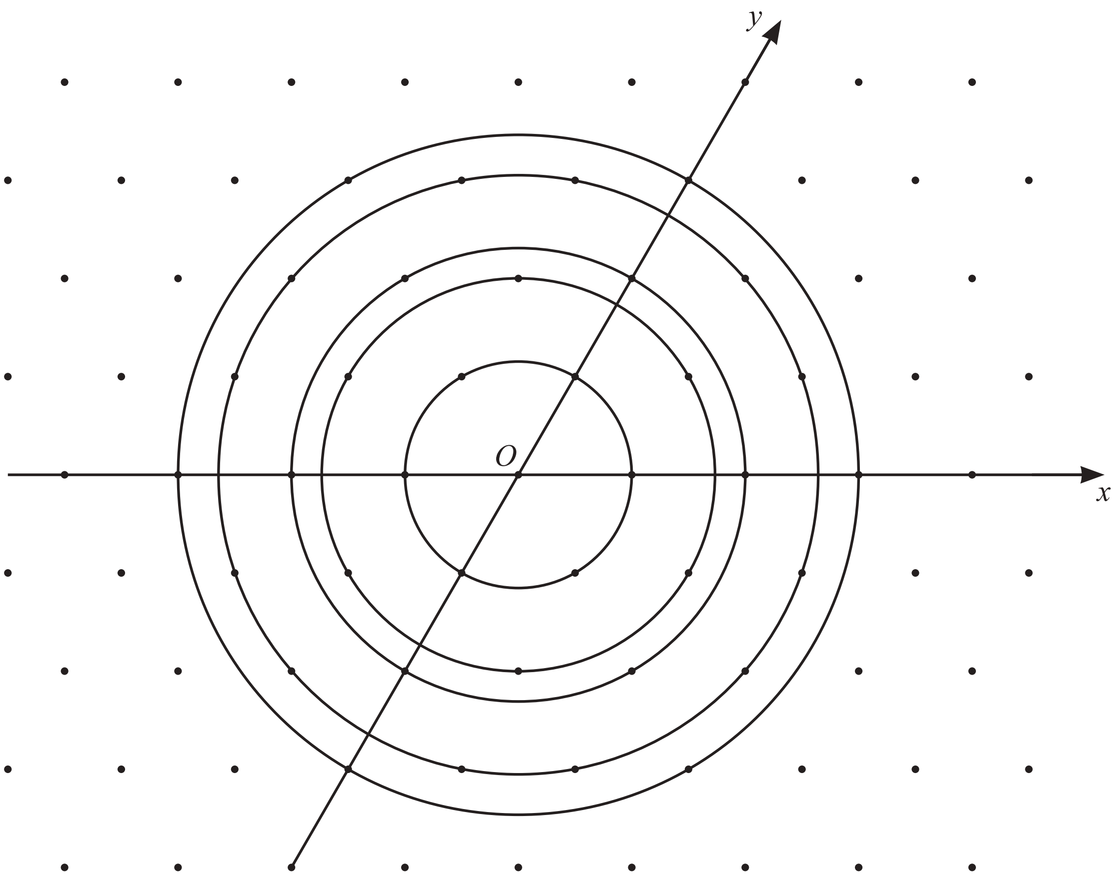
.... [2]

(c) A point on a triangular grid has coordinates (a, b), where a and b are positive integers.

Show that its distance, d, from (0, 0) is given by $d = \sqrt{a^2 + b^2 + ab}$.

[2]

(d) Use the formula in **part (c)** to calculate the distance of a 6th nearest neighbour from (0, 0). Give your answer as a square root.



.... [3]

(e) Use the formula $d = \sqrt{a^2 + b^2 + ab}$ to show that it is not possible for a nearest neighbour to be a distance of $\sqrt{14}$ from (0, 0) on a triangular grid.

[4]

B MODELLING (QUESTIONS 6 TO 9)

HIGH RIVER FLOW (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task looks at estimating the probability of high river flows.

Planners need to estimate the probability of extreme events such as high river flow.
To do this they use the return period, R years.
This is an estimate of the number of years before they expect another high river flow.

A model for R is

$$R = \frac{n+1}{h}$$

where

n is the number of years for which there are river flow results,
h is the number of times that high river flow occurs during the n years.

The probability of high river flow occurring in any year is $\frac{1}{R}$.

Example
On 3 occasions during a period of 65 years, a river flow was greater than 4000 cubic metres per second (m^3/s).

So, the return period for a water flow greater than $4000\,m^3/s$ is

$$R = \frac{65+1}{3} = \frac{66}{3}$$
$$= 22 \text{ years.}$$

This is not very likely in a year, but very likely in the lifetime of a bridge designed to last for 100 years.

The probability of a river flow greater than $4000\,m^3/s$ in any year is $\frac{1}{22}$.

6 These are the maximum flows, in m^3/s, each year for a small river.

30	119	13	30	9	64	19	83	71	64	37	61
55	33	131	91	78	59	32	29	121	7	56	65

(a) Calculate the return period, R, for a flow greater than $100\,m^3/s$.

.... [2]

(b) Use your answer to **part (a)** to find the probability of a flow greater than $100\,m^3/s$ the next year.

.... [1]

7 **(a)** The probability of an event occurring in one year is p.

(i) Write an expression, in terms of p, for the probability of the event not occurring in one year.

.... [1]

(ii) Find an expression, in terms of p, for the probability of the event **not occurring** for 10 consecutive years.

.... [1]

(iii) Explain why the probability of the event occurring at least once in 10 consecutive years is

$$1-(1-p)^{10}.$$

.... [1]

(b) A model for the probability, y, of a flow greater than $130\,\text{m}^3/\text{s}$ occurring in the river in **Question 6** at least once in x years is $y = 1-(1-0.04)^x$.

(i) Sketch the graph of $y = 1-(1-0.04)^x$ for $0 \leqslant x \leqslant 25$.

[3]

(ii) Find the number of years that pass before the probability of a flow greater than $130\,\text{m}^3/\text{s}$ first becomes greater than 50%.

.... [2]

8 The table shows the probabilities of various river flows in a year at Waverly on the Missouri River in the USA.

Flow in thousands of m^3/s (F)	Probability of flow $>F$ (p)
2	0.99
4	0.85
6	0.56
8	0.26
10	0.11
12	0.04
14	0.02
16	0.02
18	0.02
20	0.02
22	0.01

(a) A possible model for the above data is $p = 1 - kF$ where k is a constant.

Use figures from the table to decide whether or not this is a reasonable model.

.... [2]

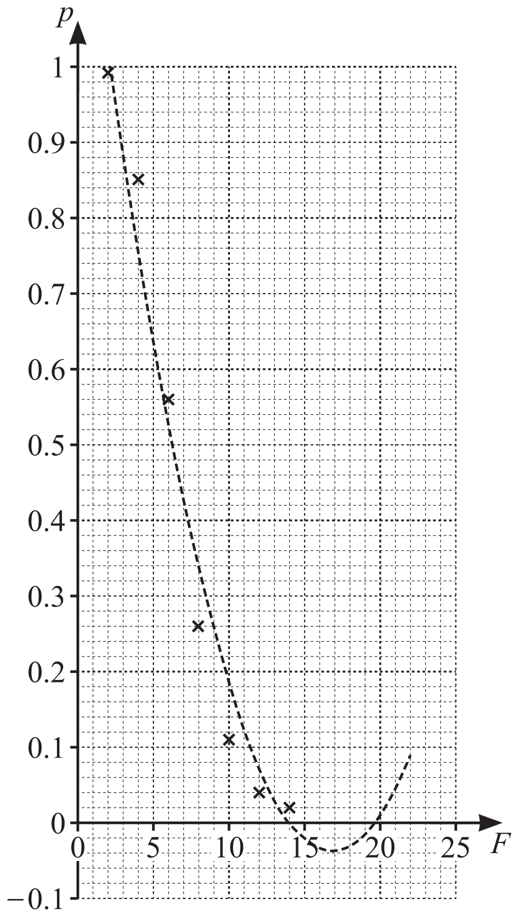
(b) Another model is $pF = c$ where c is a constant.

Use figures from the table to decide whether or not this is a reasonable model.

.... [2]

(c) Another model for the data is $p = 0.0048F^2 - 0.1619F + 1.3281$ for $2 \leqslant F \leqslant 22$.
The dashed line on the diagram shows the graph of this model.

(i) Plot the values from the table on the previous page onto the grid.
The first 7 points have been done for you.



[1]

(ii) Give the ranges of values of F for which $p = 0.0048F^2 - 0.1619F + 1.3281$ is a good model and not a good model.

A good model ....

Not a good model ....
[2]

9 Engineers use this formula to model the probability of high river flows.

$$p = 3^{-\left(\left(\frac{F}{b}\right)^{a}\right)}$$

where p is the probability of an event greater than F and
a and b are both constants.

(a) Show that $p = \frac{1}{3}$ when $b = F$.

[2]

(b) Give a reason why 8 is the best integer estimate for b when $p = \frac{1}{3}$.

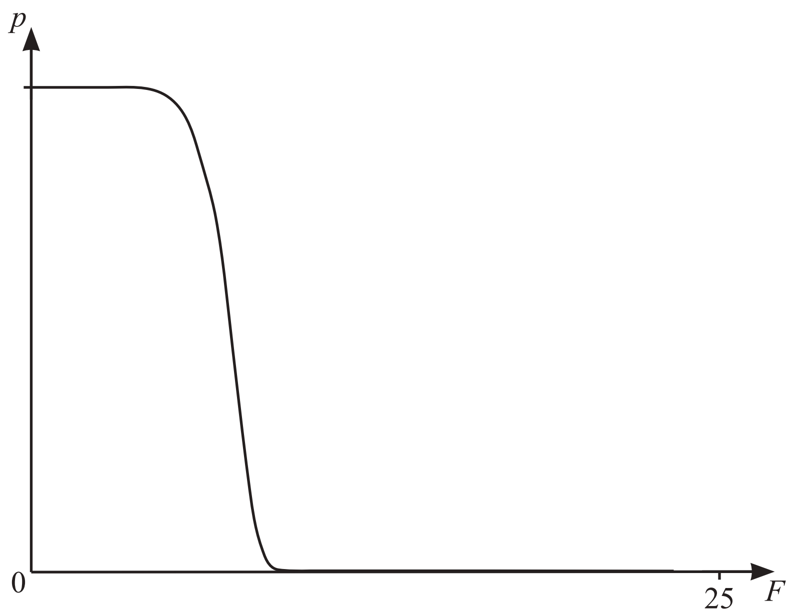
Use **part (a)** and values from the table for the Missouri River on page 12 in your answer.

.... [1]

(c) **(i)** On the axes below, sketch three graphs of $p = 3^{-\left(\left(\frac{F}{b}\right)^{a}\right)}$ for $0 \leqslant F \leqslant 25$.
Use $b = 8$ and

- $a = 0.5$,
- $a = 1$,
- $a = 2.5$.

The graph with $b = 8$ and $a = 10$ is sketched for you.



[4]

(ii) Find the coordinates of the points of intersection of the curves.

.... [2]

(iii) Describe the effect on the graph when *a* increases and the flow is between 6000 m^3/s and 8000 m^3/s.

.... [1]

(iv) Which value of *a* gives the best model for the Missouri River data on page 12?
Give a reason for your answer.

a = ... because ....

.... [2]