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CAMBRIDGE INTERNATIONAL MATHEMATICS

0607/61

Paper 6 Investigation and Modelling (Extended)

May/June 2021

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 6) and part **B** (Questions 7 to 10).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 TO 6)

ROLLING SQUARE (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at the path of a point on a square as it rolls along the x -axis.

A square of side 1 cm rolls along the x -axis.
One roll is a turn of 90° clockwise about its bottom right corner.

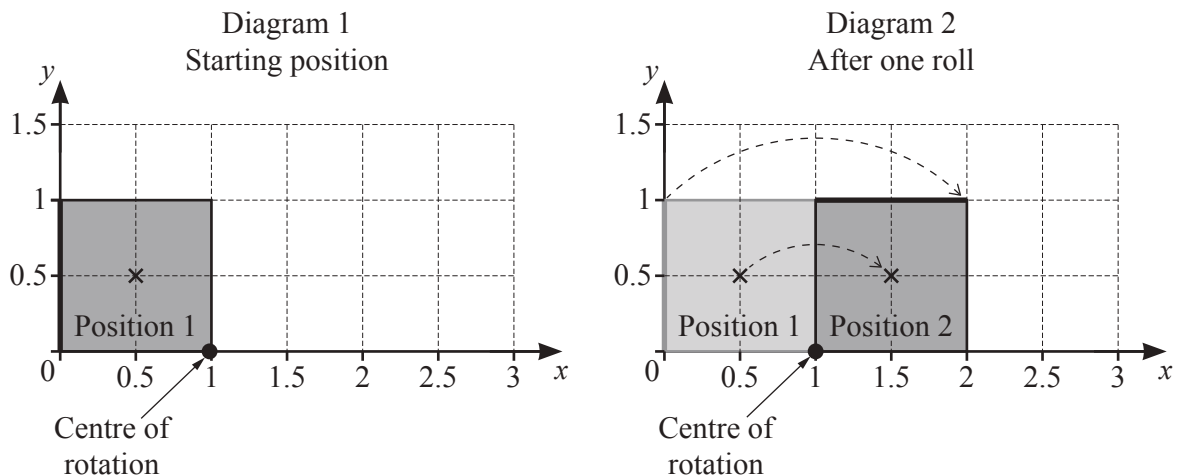
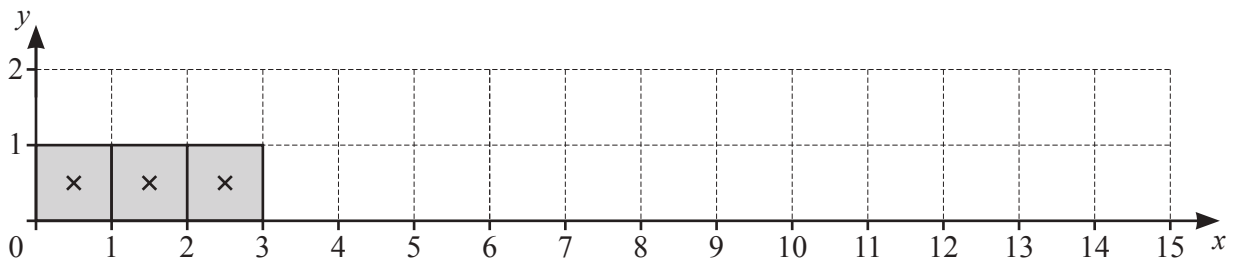


Diagram 1 shows the square in Position 1.
One side of the square is bold to help show the rotation.
The centre of the square is $(0.5, 0.5)$.

Diagram 2 shows the square rolled 90° clockwise about $(1, 0)$ to Position 2.

- 1 To get to Position 3 the square rolls 90° clockwise about $(2, 0)$.
To get to Position 4 the square then rolls 90° clockwise about $(3, 0)$.

(a) On the diagram below, draw the square in Position 4, Position 5 and Position 6.



[1]

(b) Complete this table to show the x -coordinate of the centre of the square in each position. You may use the diagram on page 2 to help you.

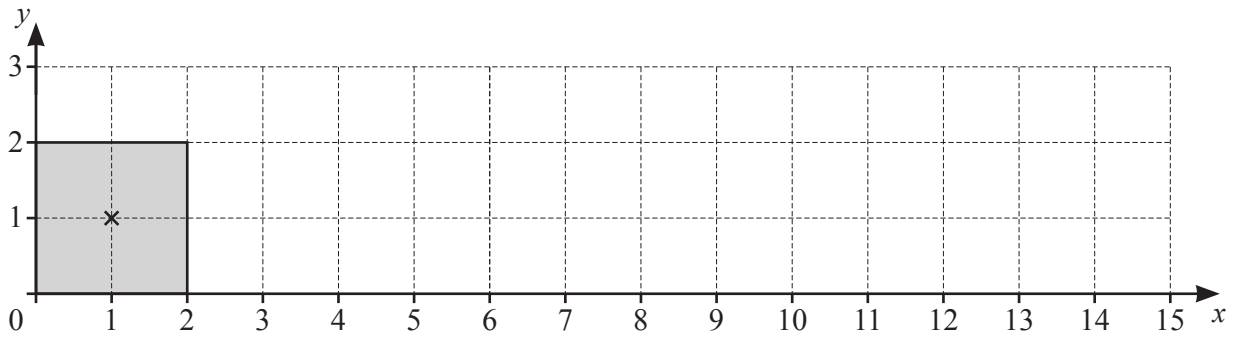
Position (n)	1	2	3	4	5	6		n
x -coordinate	0.5	1.5	2.5					

[2]

(c) Find the x -coordinate of the centre of the square in Position 92.

..... [2]

2 The side of the square is now 2 cm.



The square rolls along the x -axis in the same way as in **Question 1**.

(a) Complete the table of x -coordinates of the centre of the square in different positions.

Position (n)	1	2	3	4	5	6		n
x -coordinate	1	3						

[3]

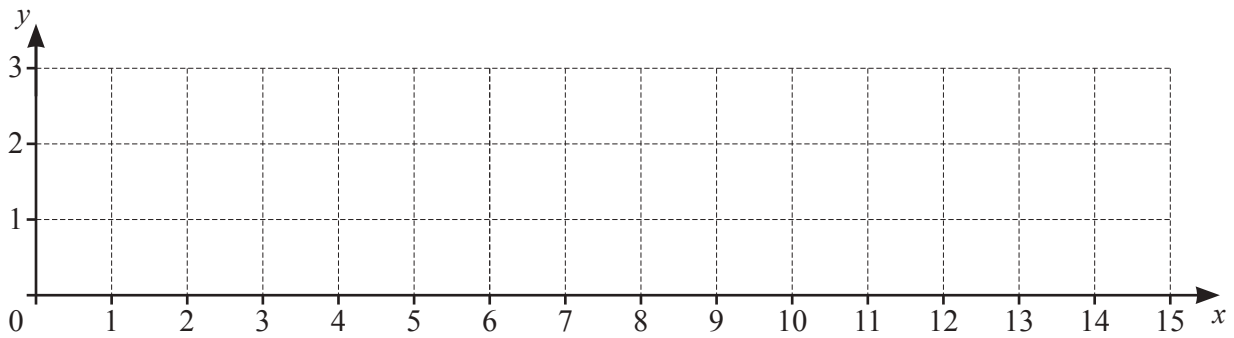
(b) Find the coordinates of the centre of the square in Position 35.

(.....,) [1]

- 3 The side of the square is now 3 cm.

Complete the table of x -coordinates of the centre of the square in different positions.
You may use the diagram below to help you.

Position (n)	1	2	3	4	5	6		n
x -coordinate	1.5							

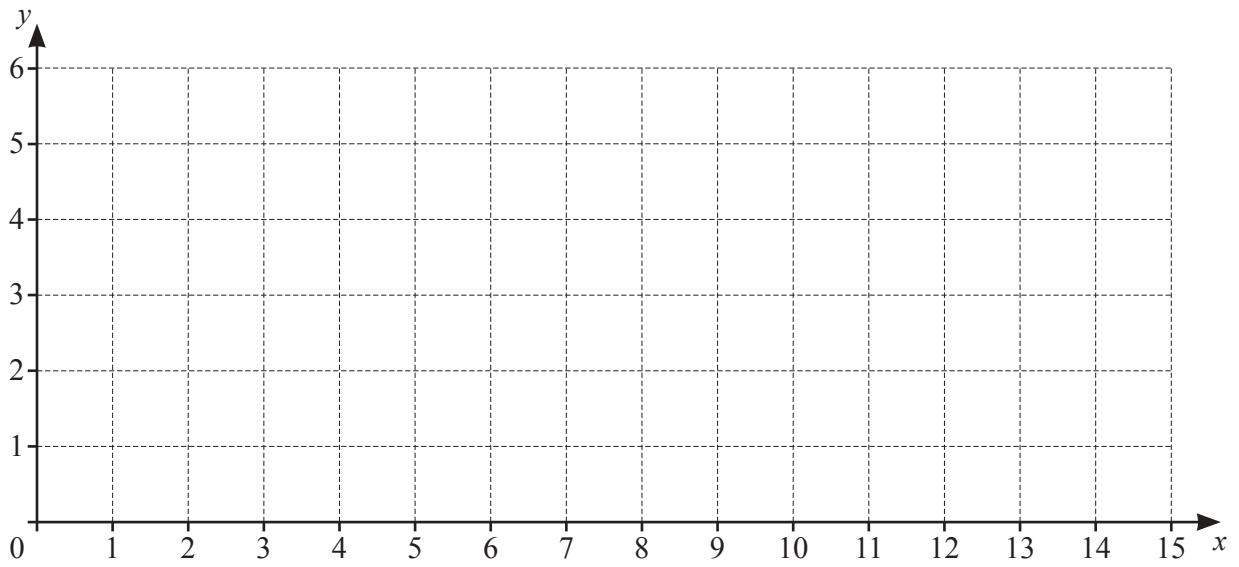


[3]

- 4 Write your expressions from **Questions 1(b), 2(a) and 3** in the table below.
Complete the table using any patterns you notice.

You may use the grid on page 5 to help you.

Side of square (w cm)	x -coordinate in Position n
1	
2	
3	
4	
5	
w	



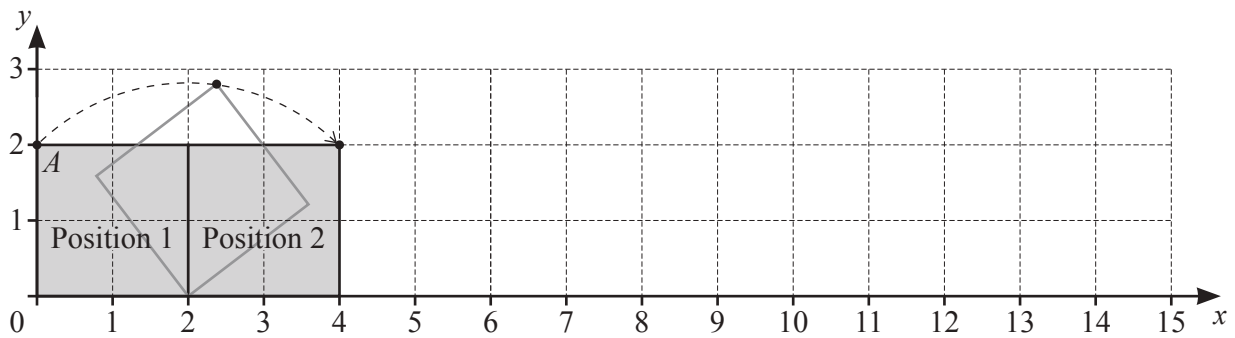
[5]

- 5 A square of side w cm rolls from Position 1 to Position 120.
At Position 120, the x -coordinate of the centre of the square is 2151.

Find the value of w .

..... [3]

- 6 A square of side 2 cm rolls along the x -axis.



- (a) The table shows the x -coordinate of the point A for each position. Complete the table.

Position	1	2	3	4	5	6	7	8	9	10	11	12	13
x -coordinate	0	4	6		8	12		14		20		22	24

[1]

- (b) In the row of x -coordinates there are four sequences.

For Positions 4, 8, 12, ... the expression for the position is $4a$, where a is a positive integer. Complete the table.

a	1	2	3	4	5		a
Position ($4a$)	4	8	12	16	20		$4a$
x -coordinate		14	22				$8a - \dots\dots\dots$
Position ($4a - 1$)	3	7	11	15	19		$4a - 1$
x -coordinate	6						$\dots\dots\dots$
Position ($4a - 2$)	2	6	10	14	18		$4a - 2$
x -coordinate	4	12	20				$\dots\dots\dots$
Position ($4a - 3$)	1						$4a - 3$
x -coordinate	0						$\dots\dots\dots$

[5]

(c) The 2 cm square rolls to Position 523.

Use **part (b)** to help you find the coordinates of point *A*.

(.....,) [4]

B MODELLING (QUESTIONS 7 TO 10)**WIND TURBINES (30 marks)**

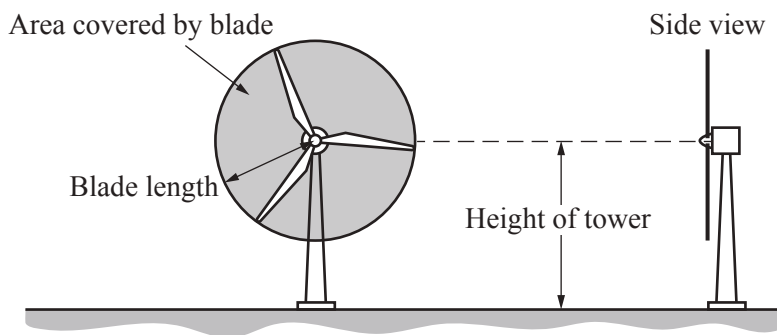
You are advised to spend no more than 50 minutes on this part.

This task looks at the use of wind turbines to generate electricity.

Area, A , of circle, radius r . $A = \pi r^2$

Circumference, C , of circle, radius r . $C = 2\pi r$

7 This is the front view and the side view of a wind turbine.



Wind turbines with longer blades generate more electrical power than wind turbines with shorter blades. Power is measured in kilowatts (kW).

A wind turbine has blades that are 27 m long and a tower of height 80 m.

(a) Find the greatest and least height above the ground for the tip of a blade as it turns.

Greatest height

Least height [2]

(b) An international soccer pitch is a rectangle measuring 70 m by 105 m.

- (i) Find the area covered by the blades of this wind turbine.
Write your answer as a percentage of the area of the international soccer pitch.

..... [3]

- (ii) New wind turbines have blades that are 107 m long.

Find the area covered by these blades as a percentage of the area of the international soccer pitch.

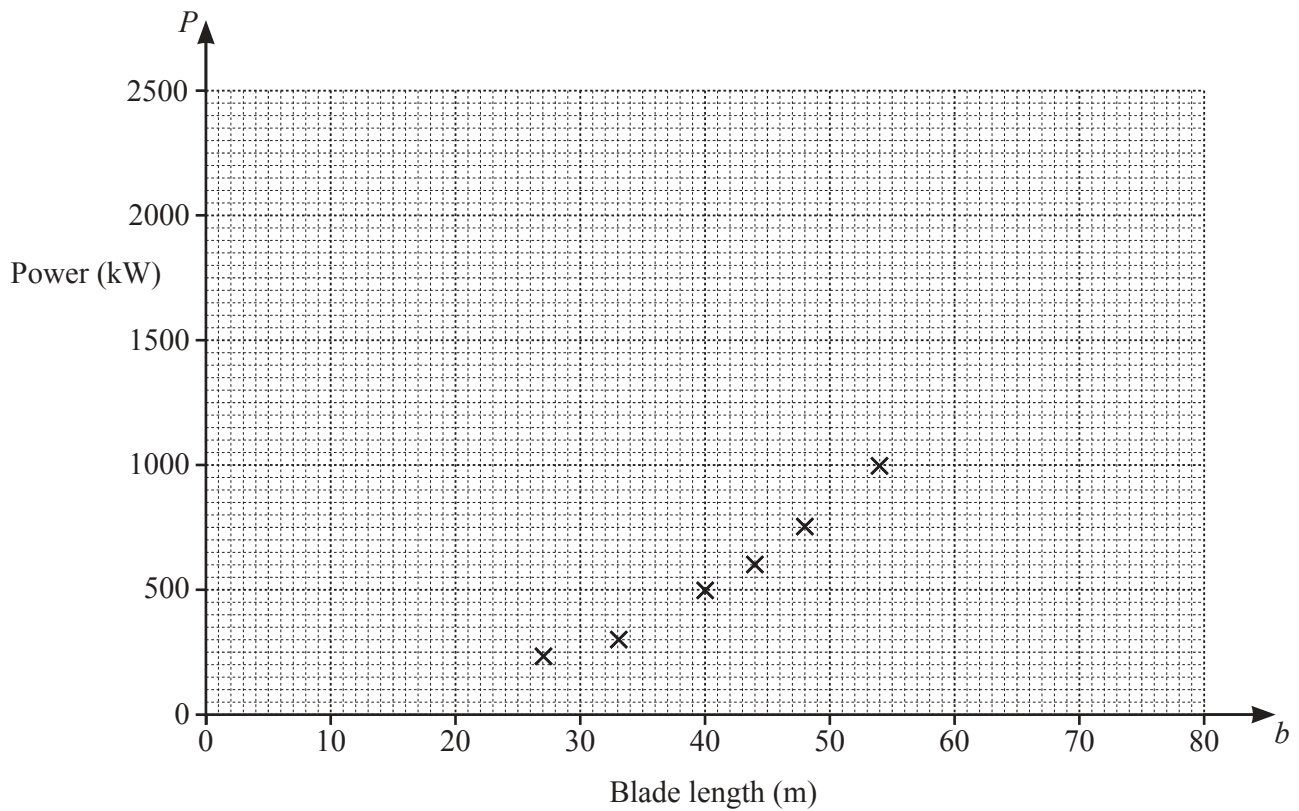
..... [2]

- 8 The amount of power generated depends on wind speed as well as the area covered.

This table shows the power (in kW) for blades of different lengths at a fixed wind speed.

Blade length (b metres)	27	33	40	44	48	54	64	72	80
Power (P kW)	225	300	500	600	750	1000	1500	2000	2500

- (a) Plot the last three points on this graph.
The first six points have been plotted for you.



[1]

(b) A model for the power, P kW, is

$$P = cb^2, \text{ where } b \text{ is the length of the blade in metres and } c \text{ is a constant.}$$

Use the information to find a value for c and write down the model.

..... [2]

(c) Another wind turbine generates 1200 kW.

Use your model to find the length of its blade.

..... [2]

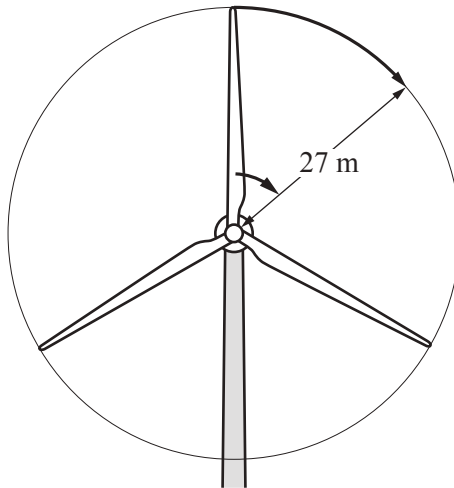
9 (a) A blade rotates through 30° every second.

- (i) Find the time it takes to complete a full turn and the number of complete turns it makes in a minute.

Time =

Number of turns = [3]

- (ii) Different parts of the blade travel through air at different speeds.



Show that the speed of the tip of this blade, with length 27 m, is 14.1 m/s, correct to 3 significant figures.

[3]

- (b) The blade with length 27 m now rotates through 40° every second.

Find the new speed of the blade tip in m/s.

..... [2]

- (c) A blade turns through t degrees every second.
The length of the blade is L metres.

Write a model for the speed, S m/s, of the blade tip in terms of π , t and L .
Give your answer in its simplest form.

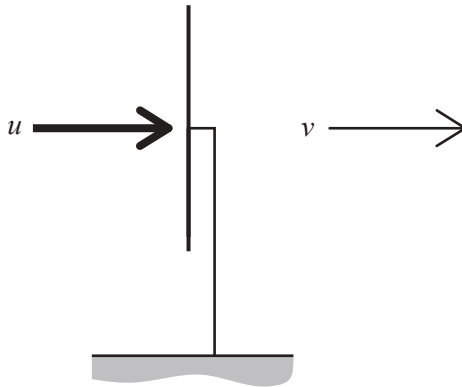
..... [2]

- (d) The maximum speed for a blade tip is 72 m/s.

Find the maximum speed of rotation, in degrees per second, for a blade with length 107 m.

..... [3]

- 10 Wind enters a turbine at a speed of u m/s.
The wind leaves the turbine at a reduced speed of v m/s.



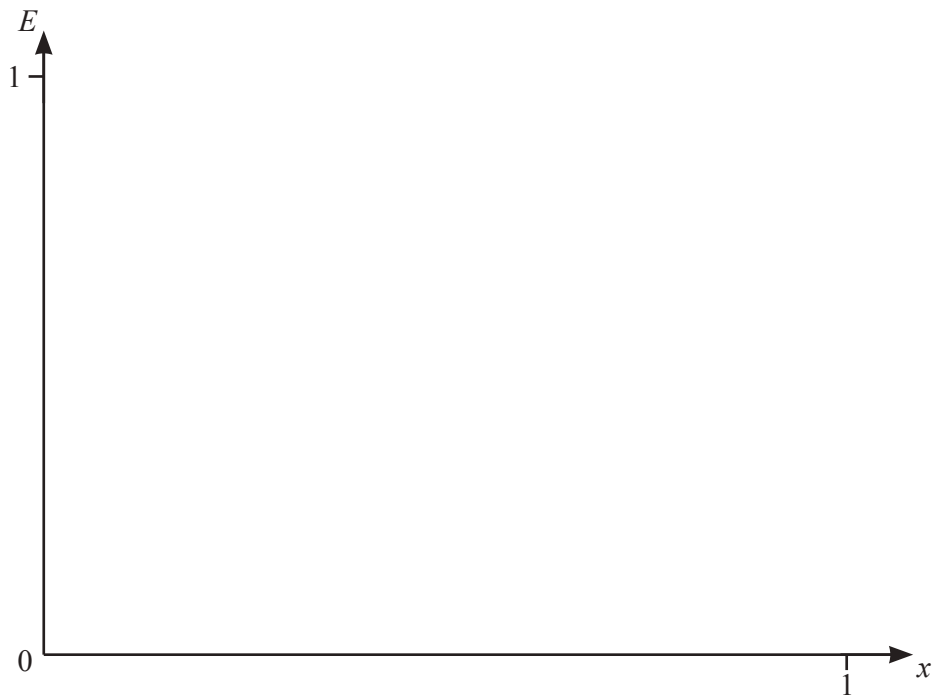
x is the fraction that v is of u , so $x = \frac{v}{u}$.

A model for the efficiency, E , of the wind turbine is $E = \frac{(1-x^2)(1+x)}{2}$.

- (a) What can you say about the wind speeds v and u if the efficiency, E , is zero?

..... [1]

(b) Sketch the graph of the model for E for $0 \leq x \leq 1$.



[2]

(c) Find the value of x that gives maximum efficiency.

..... [1]

(d) Find the greatest value for E .
Give your answer as a percentage.

..... [1]