Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER				

0002252590

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/63**

Paper 6 Investigation and Modelling (Extended) **May/June 2023**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 8) and part **B** (Questions 9 to 12).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

Answer both parts **A** and **B**.

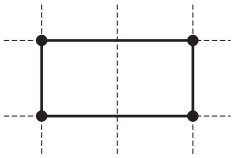
A INVESTIGATION (QUESTIONS 1 to 8)

AREA OF A PARALLELOGRAM (30 marks)

You are advised to spend no more than 50 minutes on this part.

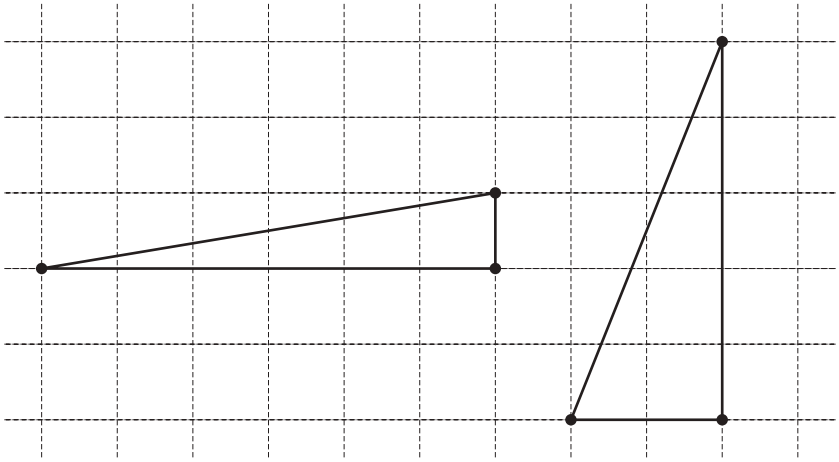
This investigation looks at the area of a parallelogram drawn on a unit square grid.

1



Find the area of the rectangle.
Write your answer inside the rectangle. [1]

2



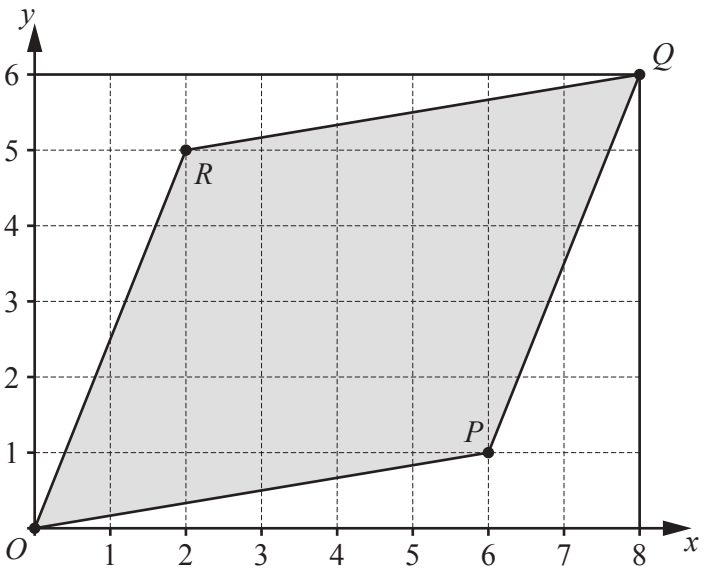
Find the area of each right-angled triangle.
Write your answer inside each triangle.

[2]

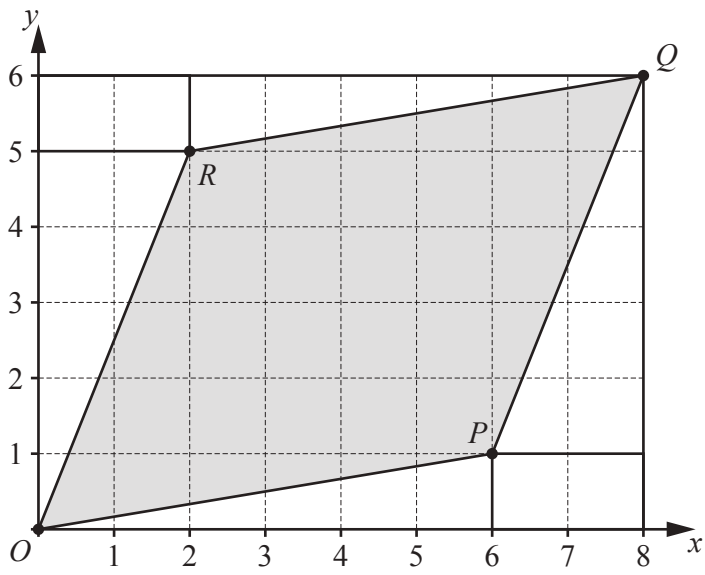
3 Throughout the rest of this task vertices are labelled anticlockwise starting at $O(0, 0)$.

These steps are the start of a method to find the area of the parallelogram $OPQR$.

Step 1 Draw a rectangle around the parallelogram.



Step 2 Fill the space between the rectangle and the parallelogram with rectangles or right-angled triangles.



(a) Find the area of the rectangle that goes around the parallelogram.

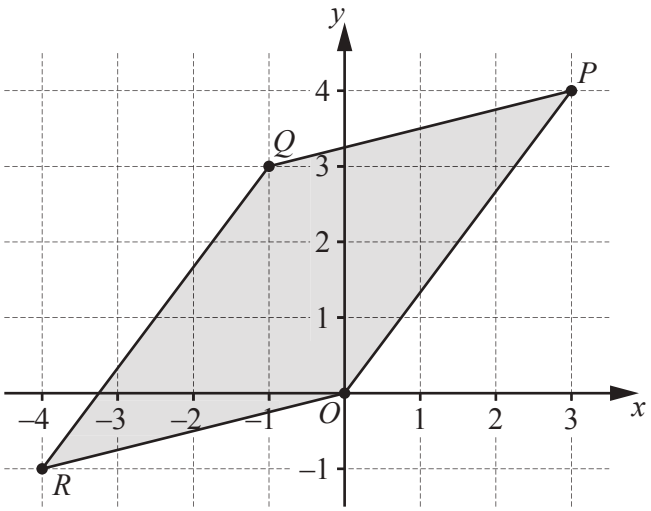
..... [1]

(b) (i) Use the results in **Question 1** and **Question 2** to write the areas of the small rectangles and right-angled triangles inside each shape. [1]

(ii) Use your answer to **part (a)** to show that the area of the parallelogram is 28.

[1]

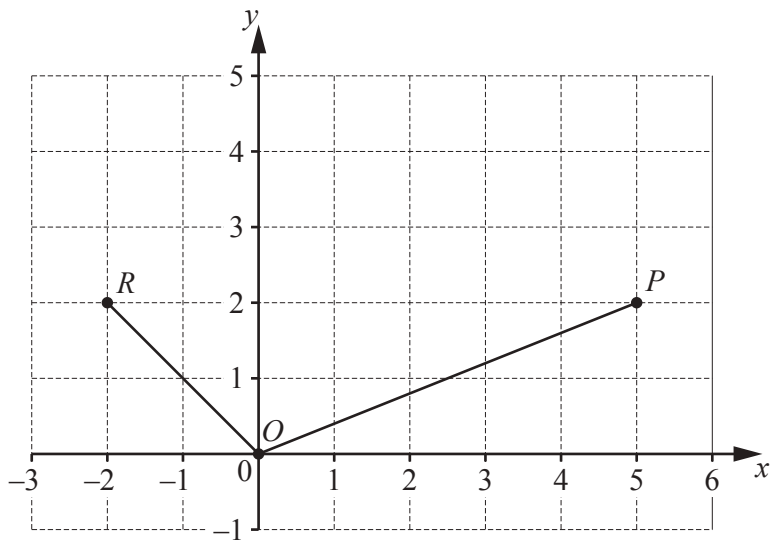
4



Use the method of **Question 3** to find the area of parallelogram *OPQR*.

..... [4]

5 **(a)**



On the diagram, complete parallelogram *OPQR*. [1]

(b) Use the method of **Question 3** to find the area of parallelogram *OPQR*.

..... [3]

6 In parallelogram $OPQR$, O is (0, 0), P has coordinates (a, b) and R has coordinates (c, d).

(a) Complete the table using your answers to **Question 4** and **Question 5** and any patterns you notice.

	$P(a, b)$	Q	$R(c, d)$	ad	bc	Area of parallelogram $OPQR$
Question 3	(6, 1)	(8, 6)	(2, 5)	30	2	28
Question 4	(3, 4)	(–1, 3)	(–4, –1)			
Question 5	(5, 2)		(–2, 2)			
	(9, 1)		(1, 8)			71
	(3, –1)			18	–4	

[5]

(b) Using the table, write down, in terms of a, b, c and d,

(i) the coordinates of Q

(..... , ...) [1]

(ii) the area of parallelogram $OPQR$.

..... [1]

7 In parallelogram $OPQR$, O is (0, 0) and P is (5, 2).
The coordinates of Q and R are positive integers.
The area of the parallelogram is 12.

Use your answers to **Question 6(b)** to find three possible parallelograms.
Write the coordinates of the points Q and R in each case.

Q(..... , ...) R(..... , ...)

Q(..... , ...) R(..... , ...)

Q(..... , ...) R(..... , ...) [4]

8 **(a)** Draw a sketch showing that two congruent triangles make a parallelogram.

[1]

(b) The integers g and h are between –50 and 50.

The vertices of triangle OGH are $O(0, 0)$, $G(7, g)$ and $H(h, 8)$.
The area of the triangle is 25.

Find how many triangles are possible.

..... [4]

B MODELLING (QUESTIONS 9 to 12)

FORGETTING CURVES (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task looks at how memory decreases with time after we have memorised something.

9 Research has shown that after a day we remember $\frac{1}{2}$ of what we memorised the day before.
This decrease in memory continues for the following days.
Retention is the fraction that we still remember.

Complete the retention at the end of day d as a power of 2.

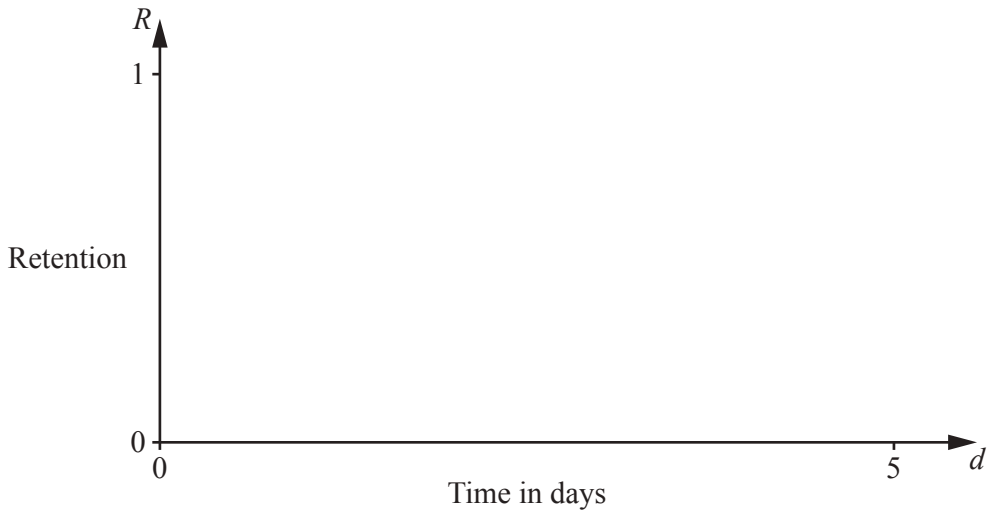
Time	At the start	At the end of					
		day 1	day 2	day 3	day 4	.....	day d
Retention	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	.....	$2^{\text{.....}}$

[1]

10 Some people have more powerful memories than other people.
A possible model for the retention, R, is

$R = 2^{\frac{-d}{p}}$, where d is the time in days and p is the power of a person's memory.

(a) On the grid sketch the graphs of R for
- $p = 1$
- $p = 5$.



[3]

(b) Find the difference in the retention, R, for $p = 1$ and $p = 5$ at the end of 5 days.
Give your answer correct to two decimal places.

..... [2]

(c) When $p = 5$, the graph is nearly a straight line.
A model for this straight line is $R = ad + b$ where a and b are constants.

(i) Use the values of R when $d = 0$ and $d = 5$ to find the model.

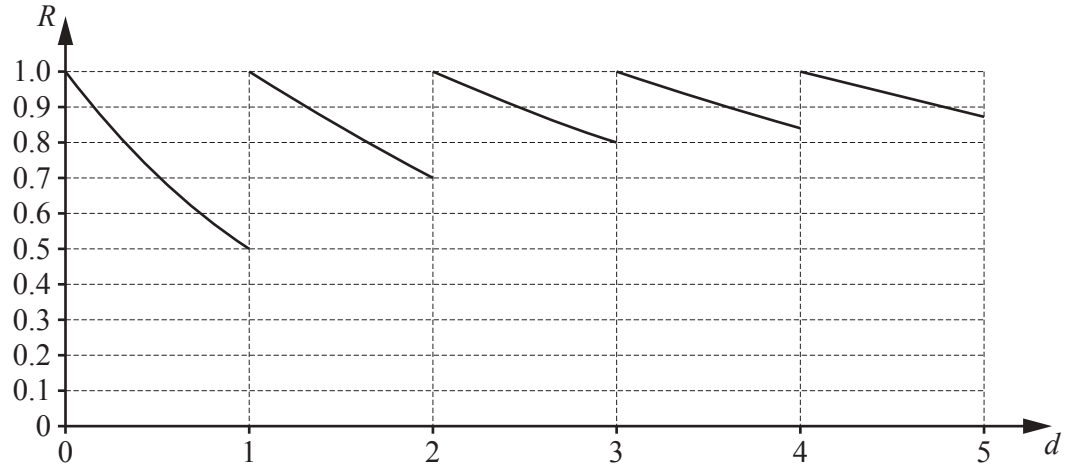
..... [3]

(ii) Make a statement about the validity of your model after more than 10 days.

.....

..... [2]

11 One method to improve your memory is to memorise the material again each day.
The graphs show the retention over 5 days when you use this method.



At the end of each day the graph is translated 1 unit to the right and the power, p, of your memory increases by 1.

The general model is $R = 2^{\frac{-d}{p}}$.

(a) (i) At the start, $p = 1$.

Write down why $p = 5$ after 4 days.

..... [1]

(ii) When a graph is translated by k units to the right, its equation changes from $y = \mathrm{f}(x)$ to $y = \mathrm{f}(x-k)$.

Show that $R = 2^{\frac{4-d}{5}}$ gives the retention after 4 days.

[1]

(b) At the end of 1 day the retention is $\frac{1}{2}$, as seen on the graph.
After 5 days there is no more memorising again and $R = 2^{\frac{4-d}{5}}$.

Find the next time, d, that the retention, R, is $\frac{1}{2}$.

..... [2]

12 The German psychologist Hermann Ebbinghaus did important research into retention. He learned a list of nonsense words and measured how much he could remember at the end of certain times.

The table shows his results.

	S	*T*	*U*	*V*	*W*	*X*	*Y*	*Z*
Time since learning	0 minutes	20 minutes	1 hour	9 hours	1 day	2 days	6 days	1 month (31 days)
Retention	1	0.58	0.44	0.36	0.33	0.28	0.25	0.21

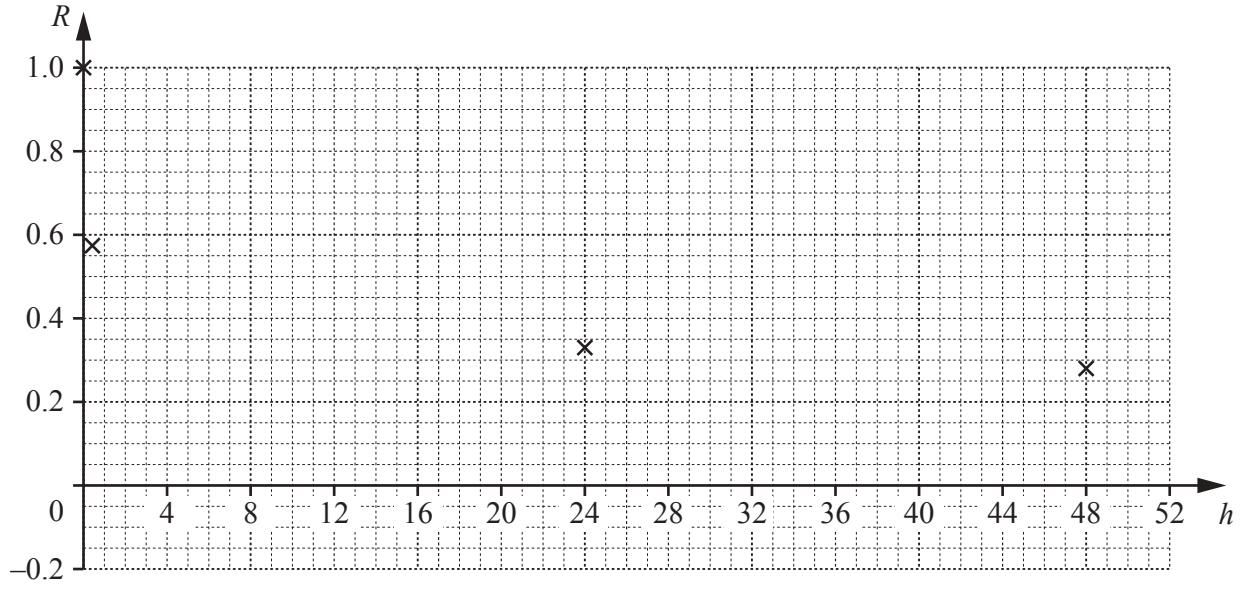
(a) Complete the table by changing the time to hours.

	S	*T*	*U*	*V*	*W*	*X*	*Y*	*Z*
Time in hours since learning (*h*)	0		1	9	24	48		744
Retention (*R*)	1	0.58	0.44	0.36	0.33	0.28	0.25	0.21

[1]

(b) The graph shows the points *S*, *T*, *W* and *X* from the table in **part (a)**.

Plot the points *U* and *V*.



[1]

(c) A possible model for Ebbinghaus's data is $R = a \times 2^{-h} + b$.

(i) Use points S and U to write two equations in terms of a and b.

.....

..... [1]

(ii) Use the method of simultaneous equations to find the value of a and the value of b.

$a =$ .....

$b =$ ..... [3]

(iii) Write down the model and sketch it on the graph on page 12.

..... [1]

(iv) Make a statement about the validity of the model in **part (iii)**.

..... [1]

(d) In 1885 Ebbinghaus wrote that a good model for his results was

$$R = \frac{1}{k \log h + 1} \quad \text{where } k \text{ is a positive constant.}$$

(i) Give algebraic reasons why this model is not valid for $0 < h < 1$.

.....

.....

.....

..... [2]

(ii) Use the point $W(24, 0.33)$ to find the value of k correct to two decimal places.

..... [3]

(iii) Show that at the end of one month the difference between the model and Ebbinghaus's results is less than 0.05 .

[2]

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