Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5610782237

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/61**

Paper 6 Investigation and Modelling (Extended) **May/June 2023**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS
- Answer both part **A** (Questions 1 to 4) and part **B** (Questions 5 to 7).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION
- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

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Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 4)

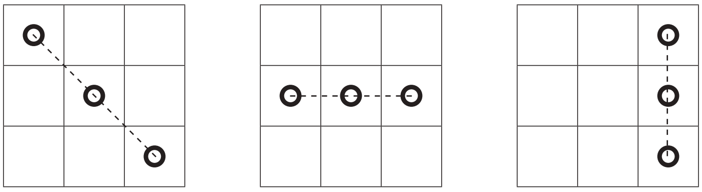
WINNING LINES (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at the number of winning lines on a grid.

1 In a game you win by making a straight line of three Os on a 3 by 3 grid.

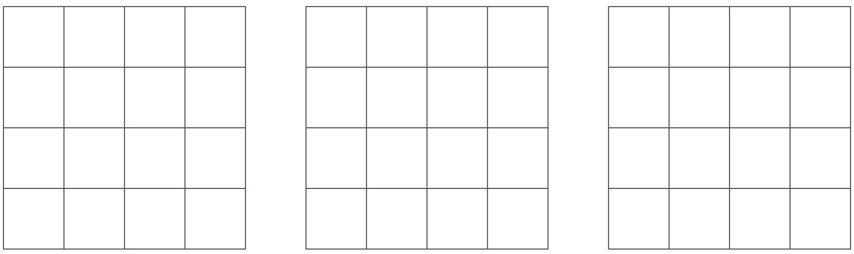
Examples of a winning line with three Os.



There are 8 different winning lines.

(a) Another grid is 4 by 4.
You now need four Os in a line to win.

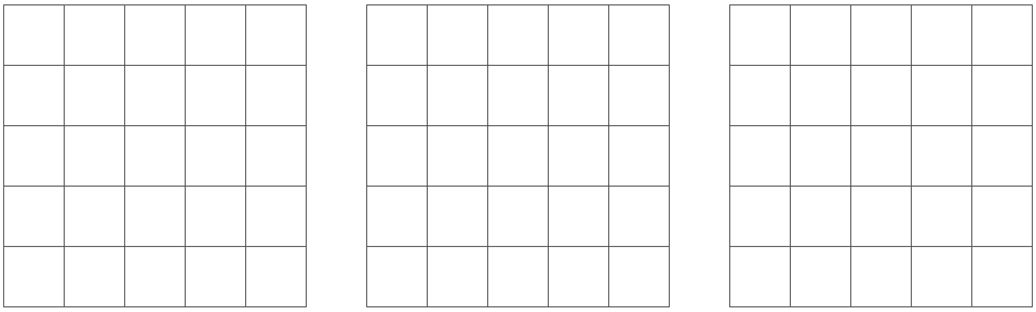
Find the number of winning lines on a 4 by 4 grid.
You may use the grids to help you.



.... [2]

(b) Another grid is 5 by 5.
You now need five **O**s in a line to win.

Find the number of winning lines on a 5 by 5 grid.
You may use the grids to help you.



.... [1]

(c) A grid is n by n.
You need n **O**s in a line to win.

Find an expression, in terms of n, for the number of winning lines.
Give your answer in its simplest form.

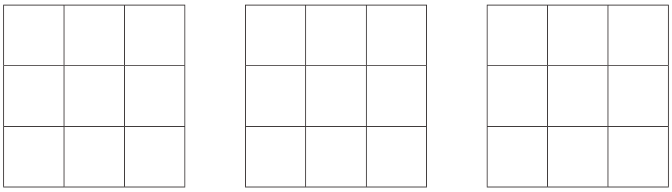
.... [2]

2 A grid is n by n.
In a different game a winning line is one **O** less than n.
To make a line, the **O**s must be in squares that are next to each other.

(a) In a 3 by 3 grid you need two **O**s in a line to win.

Complete the table to find the number of winning lines with two **O**s.
You may use the grids below the table to help you.

Size of grid	Number of winning lines			
	Horizontal	Vertical	Diagonal	Total
3 by 3			8	

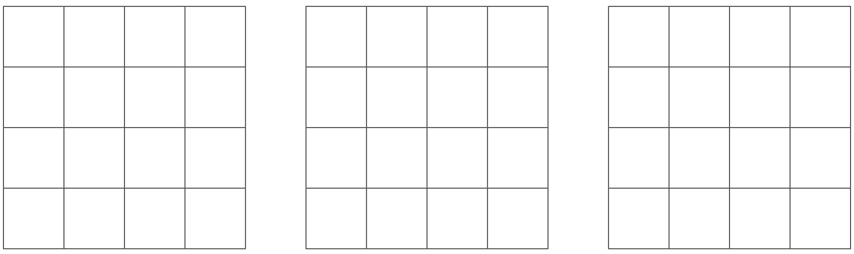


[2]

(b) In a 4 by 4 grid you need three **O**s in a line to win.

Complete the table to find the number of winning lines with three **O**s.
You may use the grids below the table to help you.

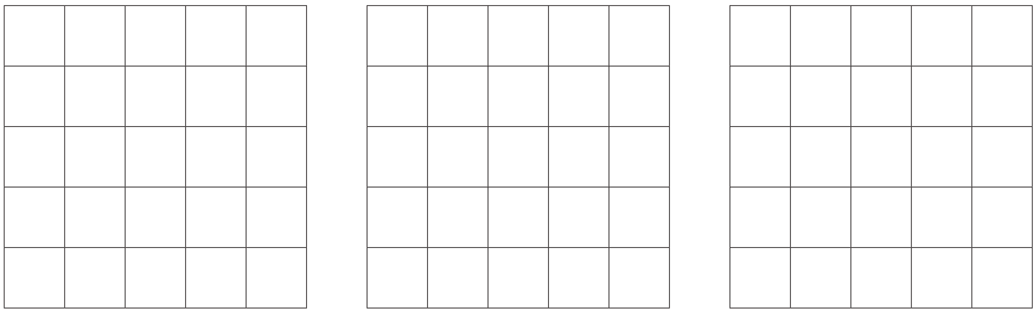
Size of grid	Number of winning lines			
	Horizontal	Vertical	Diagonal	Total
4 by 4				



[2]

(c) Copy your results from **part (a)** and **part (b)** into this table.
Complete the table.
You may use the grids below the table to help you.

Size of grid	Number of winning lines			
	Horizontal	Vertical	Diagonal	Total
3 by 3			8	
4 by 4				
5 by 5			8	
n by n				



[2]

(d) In an n by n grid you need $(n-1)$ **O**s in a line to win.
n must be at least 3.

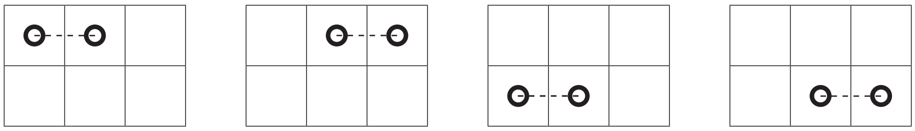
In one grid the total number of winning lines is a square number less than 50.

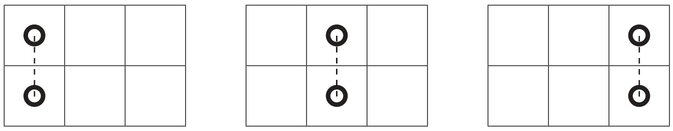
Find the grid size.

.... [3]

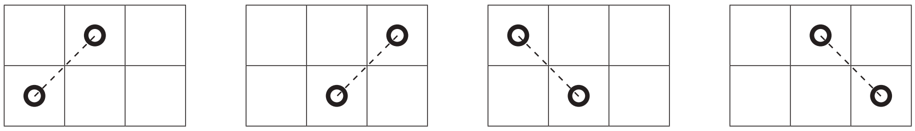
3 A rectangular grid has height 2 and width at least 2.
You need two **O**s in a line to win.

These diagrams show all the winning lines with two **O**s on a 2 by 3 grid.



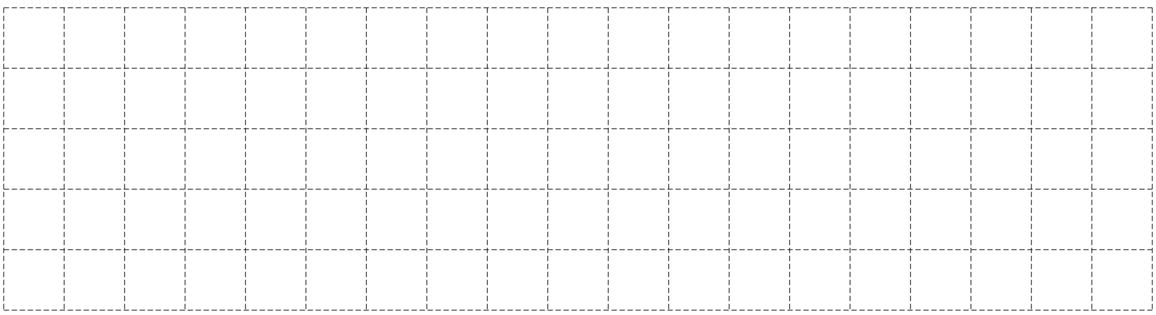


There are 11 winning lines



(a) Complete the table for the number of winning lines with two **O**s.
You may use the grid below the table to help you.

Size of grid	Number of winning lines			
	Horizontal	Vertical	Diagonal	Total
2 by 2	2	2	2	6
2 by 3	4	3	4	11
2 by 4	6	4		
2 by 5		5	8	
2 by w		w		



[3]

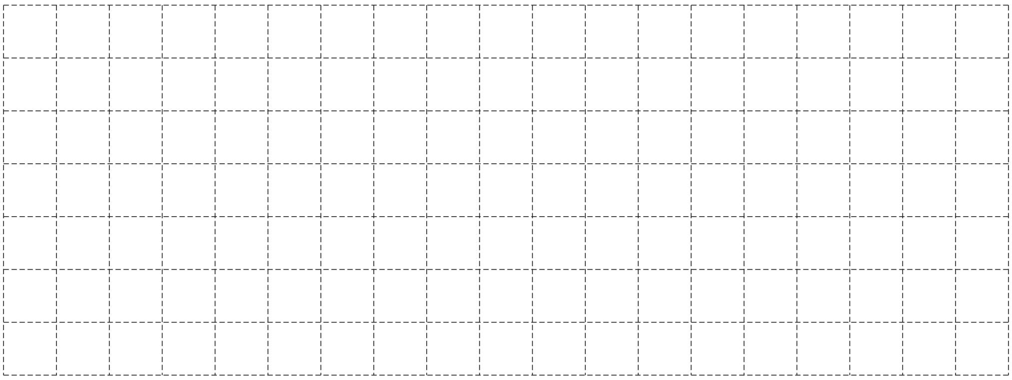
(b) A rectangular grid has height n and width w where $n \geqslant 2$ and $w \geqslant n$.
Any winning line of **O**s has n **O**s.

(i) Give a reason why there are always w vertical winning lines.

....

.... [1]

(ii) Find an expression, in terms of n and w, for the **total** number of horizontal winning lines.
You may use the grid to help you.



.... [2]

(iii) Use **part (i)** and **part (ii)** and the geometry of the grid to find an expression for the total number of winning lines.
Do not simplify your expression.

.... [1]

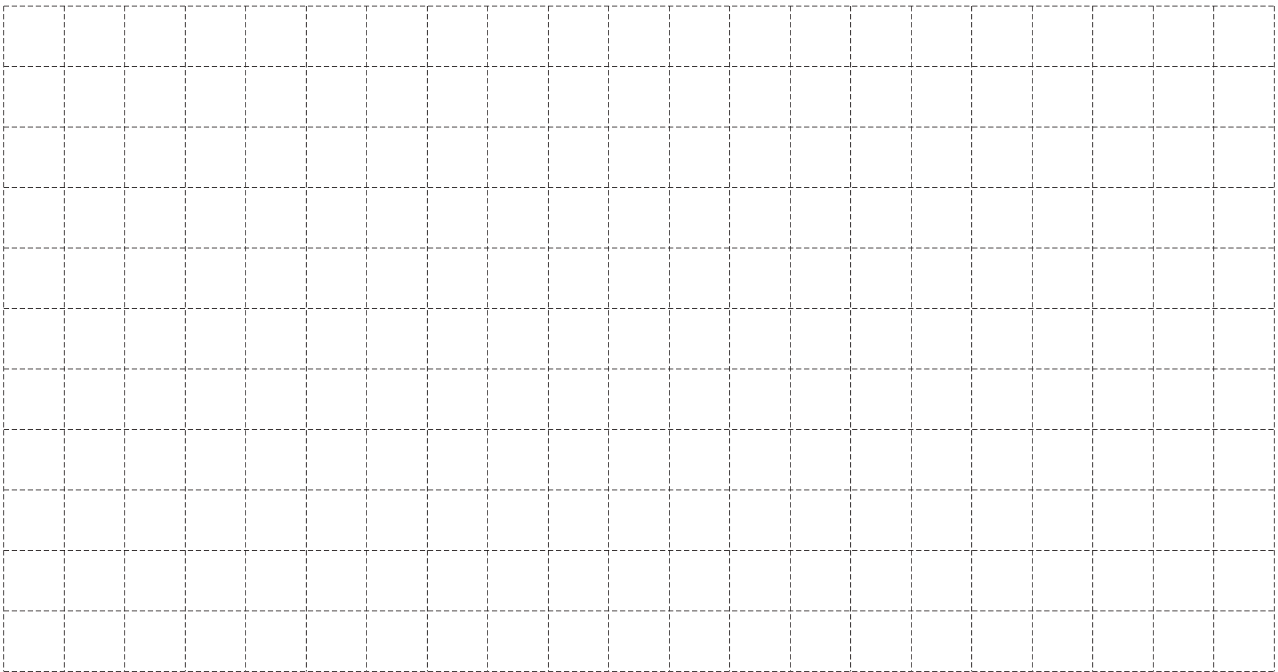
(iv) Using your answer to **part (iii)** show that, when $w = 2n$, the expression for the total number of winning lines is $n^2 + 5n + 2$.

[3]

4 A rectangular grid has height n and width w, where n must be at least 3 and $w \geqslant n$.
Any winning line of **O**s has $(n-1)$ **O**s.

(a) Complete the table with an expression for the number of diagonal winning lines in terms of n and w.
You do not need to simplify your expression.
You may use the grid below the table to help you.

Grid with width w	
Number of horizontal winning lines	$n(w-n+2)$
Number of vertical winning lines	$2w$
Number of diagonal winning lines	



[3]

(b) A grid of height 4 contains 54 winning lines with three **O** s.

Find the width of the grid.

.... [3]

B MODELLING (QUESTIONS 5 to 7)

THE SPEED OF SOUND (30 marks)

You are advised to spend no more than 50 minutes on this part.

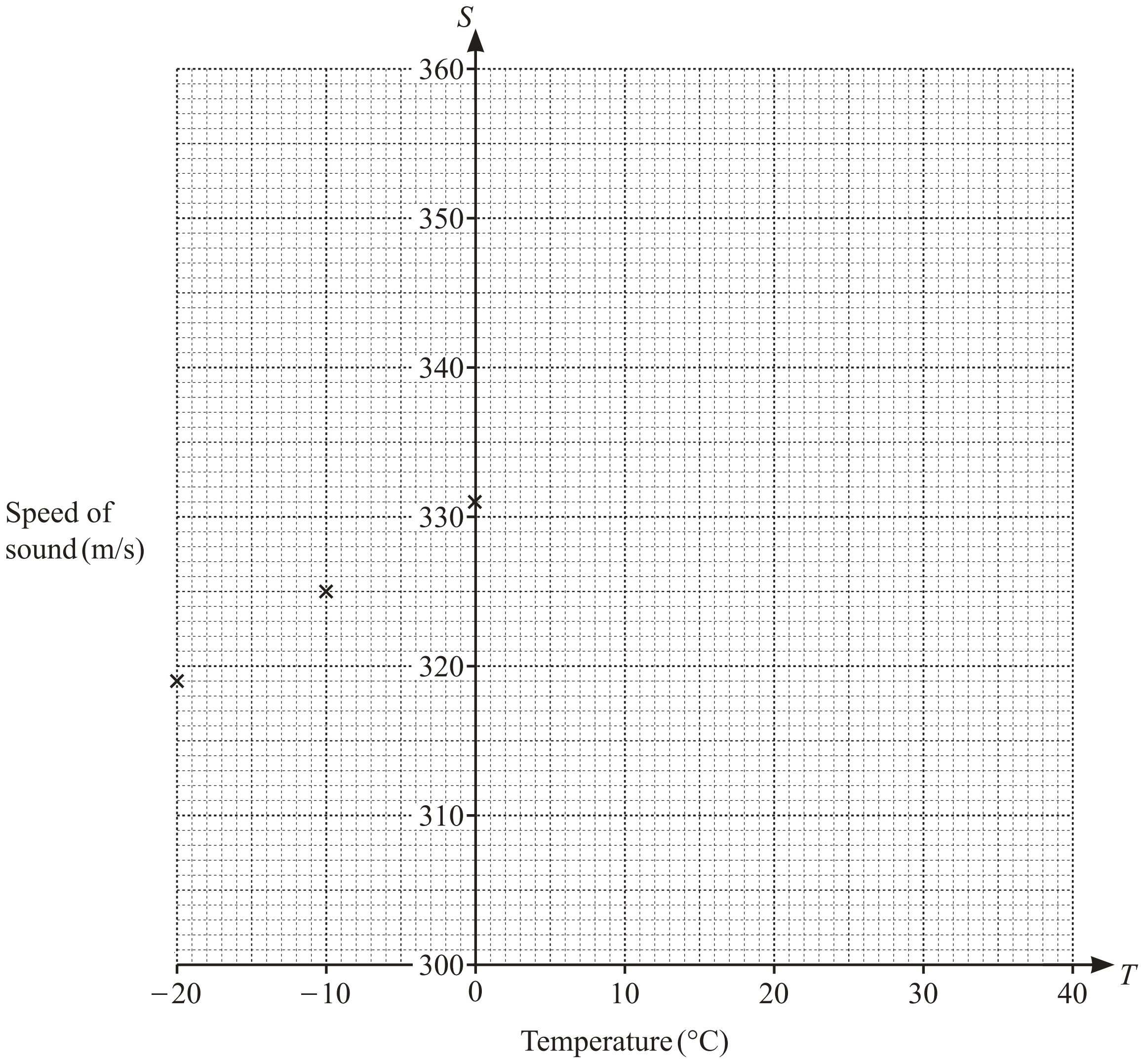
This task is about the speed of sound travelling through the air.

The speed of sound, S m/s, is often given as 343 m/s.
This means that a sound will travel 343 metres in 1 second through dry air when the temperature is 20 °C.

5 The table shows the speed of sound, correct to the nearest m/s, in dry air at different temperatures.

Temperature (T °C)	–20	–10	0	10	20	30	40
Speed of sound (S m/s)	319	325	331	337	343	349	355

(a) Plot these points on the axes below and draw a line of best fit.
The first three points have been plotted for you.



[3]

(b) Find a model for S in terms of T.

.... [2]

(c) **(i)** Find the speed of sound when the air temperature is 27 °C.

.... [2]

(ii) Ellie watches a storm approaching.
The air temperature is 27 °C.
She sees a flash of lightning as it happens and hears the sound of its thunder 5 seconds later.

Use your answer to **part (i)** to find how far the flash is from Ellie in kilometres.

.... [2]

6 Humid air contains water vapour.
Sound travels at a different speed in humid air compared to its speed in dry air.

Air with 0% humidity is dry air.
Air with 100% humidity holds all the water vapour that it can.

The table shows the speed of sound, correct to 2 decimal places, for different values of humidity, when the temperature is 20 °C.

Humidity (H%)	0	20	40	60	80	100
Speed of sound (S m/s)	343.37	343.62	343.87	344.12	344.37	344.62

(a) Show that a linear relationship connects H and S.

[2]

(b) **(i)** I is the increase in the speed of sound with humidity when the temperature is 20 °C.

Show that $I = 0.0125H$.

[1]

(ii) Assume that the increase in **part (i)** is the same for all temperatures.
When $T = 0$ and $H = 0$, the speed of sound, correct to 2 decimal places, is 331.37 m/s.

Use this information and **part (i)** to change your model for S in **Question 5(b)**.
Give your answer in the form $S = x + yT + zH$, where x, y and z are constants.

.... [1]

(c) A firework makes a loud bang.
The sound travels 1 km.
The temperature is 20 °C.

Find the difference between the time for the sound to travel 1 km in dry air and the time to travel 1 km when the humidity is 63%.
Give your answer in seconds, correct to 5 decimal places.

.... [4]

7 In **Question 6(b)(ii)** you wrote a new model for the speed of sound using $I = 0.0125H$ that assumed I was the same at all temperatures.

In fact the increase in speed, I, does change as temperature changes.

(a) This table shows values of I for some temperatures.

Temperature (T°C)	–20	0	10	40
Increase (I)	$0.0006H$	$0.0031H$	$0.0056H$	$0.0412H$

This is a new model for I.

$$I = \frac{k^T}{350}H$$

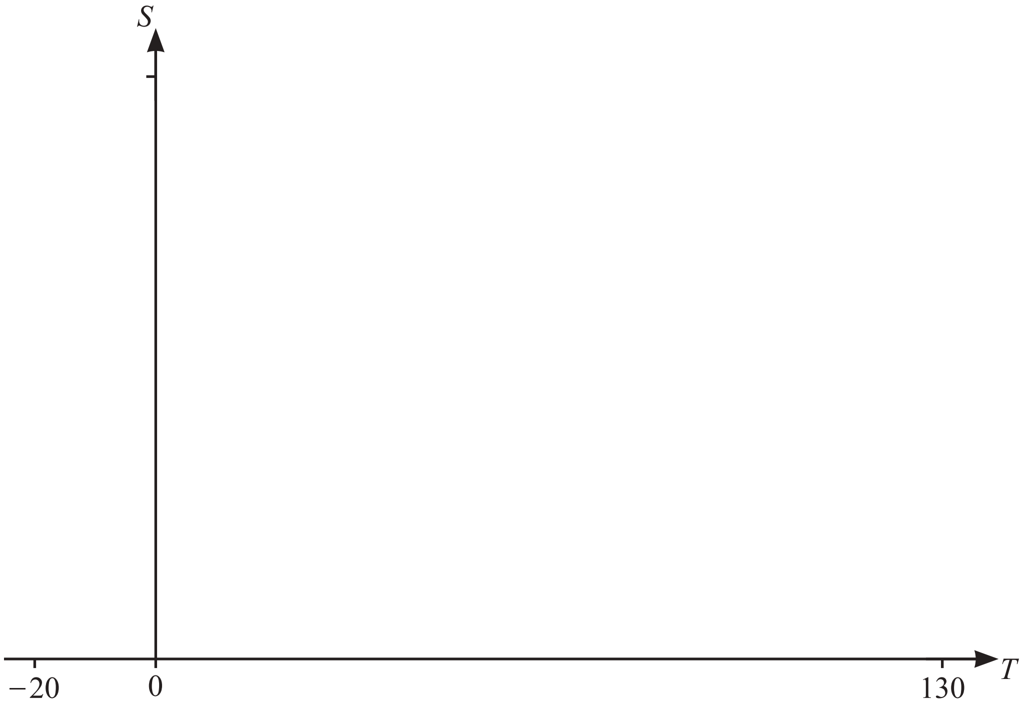
When $T = 40$, find the value of k.
Give your answer correct to 3 decimal places.

.... [3]

(b) Use **part (a)** and your answer to **Question 6(b)(ii)** to write down another model for S in terms of T and H.

.... [1]

(c) For places where humans live, air temperatures are usually between –40 °C and 50 °C but can reach above 100 °C near volcanic eruptions and wildfires.



On the axes, sketch the graph of your model when $H = 25$.

[3]

(d) There is a loud explosion in a desert at exactly 12 18.
The temperature is 45 °C and the humidity is 25%.
The explosion is heard 100 km away.

Find the time when the explosion is heard.
Give your time correct to the nearest second.

.... [6]

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