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CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/62**

Paper 6 Investigation and Modelling (Extended) **February/March 2023**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS
- Answer both part **A** (Questions 1 to 3) and part **B** (Questions 4 to 6).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION
- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 3)

SPLIT NUMBERS (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at numbers that are split into two parts.

A start number is split into two parts
the units, U and
the remaining multiple of ten, T.

<u>Examples</u>

start number	T	U
37	30	7
125	120	5
1526	1520	6

1 **(a)** Complete the table.

start number	T	U	T^2	U^2	$T^2 - U^2$	$T - U$	$\frac{T^2 - U^2}{T - U}$
125	120	5	14 400	25	14 375	115	125
34							34
	40	2					
50		0	2500			50	
		1		1	22 499		151
		0			49 000 000		

[5]

(b) What do you notice about the start number and the value of $\frac{T^2 - U^2}{T - U}$ for each row of the table in **part (a)**?

.... [1]

(c) Copy your answers from **part (a)** into the shaded columns.

Complete the table.

start number	T	U		T^2-U^2	$T+U$	$\frac{T^2-U^2}{T+U}$
125	120	5		14 375	125	115
34					34	
	40	2			42	38
50		0			50	
		1		22 499		
		0		49 000 000		

[2]

(d) Use your tables from **part (a)** and **part (c)** to complete this statement.

$\frac{T^2-U^2}{T+U} =$ .... [1]

(e) **(i)** Show that the expansion of $(T-U)(T+U)$ is T^2-U^2.

[1]

(ii) Using **part (i)** write 180^2-5^2 as the product of two integers.

[2]

2 This question is about $T^2 + U^2$.
$T + U$ and $T - U$ are not always factors of $T^2 + U^2$.

(a) Use the table to help you investigate the start numbers from 35 to 40.
Find whether $T + U$ or $T - U$ or both are factors of $T^2 + U^2$ for these start numbers.

You may not need to use all the columns.

start number	T	U	T^2	U^2					
35	30	5	900	25					
36	30	6	900	36					
37	30	7	900	49					
38	30	8	900	64					
39	30	9	900	81					
40	40	0	1600	0					

[5]

(b) A start number is a multiple of 10.

Use algebra to explain why $T + U$ and $T - U$ are always factors of $T^2 + U^2$.

[2]

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(c) **(i)** Show that $T^2 + U^2 = (T + U)^2 - 2TU$.

[1]

(ii) When $T + U$ is a factor of $T^2 + U^2$, then $T + U$ must also be a factor of $2TU$.

Explain why this statement is true.

[1]

(d) When $T - U$ is a factor of $T^2 + U^2$, then $T - U$ must also be a factor of $2TU$.

Use the expansion of $(T - U)^2$ to explain why this statement is true.

[3]

3 This question is about $T^3 - U^3$.

(a) Complete the table.

start number	$T-U$	T^3-U^3	$\dfrac{T^3-U^3}{T-U}$	T^2+TU+U^2
12	8		124	
25	15		525	
31	29		931	

[3]

(b) **(i)** Use the table to complete the statement below.

$(T-U)(T^2+TU+U^2) =$ .... [2]

(ii) Use algebra to show that the statement in **part (i)** is always true.

[1]

The modelling task starts on the next page.

B MODELLING (QUESTIONS 4 to 6)

FERRIES (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task is about the time it takes a ferry to leave an island and travel to the mainland.

Jin owns a ferry.

4 The ferry leaves the island when there are 10 passengers on the ferry.
In this question, there are already 6 passengers on the ferry.
This means the ferry needs 4 more passengers before it leaves.

Jin wants to model how long the ferry waits at the island before travelling to the mainland.
He assumes the passengers arrive one at a time.

(a) Jin uses the one-digit numbers 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 to make a rule.
This rule gives the number of minutes between each passenger arriving.

One-digit numbers	0	1, 2	3, 4, 5, 6, 7	8, 9
Number of minutes between passengers	1	2	3	4

(i) Jin uses his rule with this table of random numbers to model the time between each passenger arriving.

Numbers used for	One-digit random numbers			
Trial 1	9	3	5	0
Trial 2	2	1	7	5
Trial 3	5	8	6	0
Trial 4	2	4	9	6
Trial 5	4	1	7	9
Trial 6	9	3	5	8
Trial 7	5	1	0	8
Trial 8	3	7	5	5
Trial 9	9	5	4	6
Trial 10	3	3	1	7

Each trial models the times between passengers arriving.
Use the last two rows of the random number table to complete the table of trials.
Jin has completed the first eight trials.

<u>Example</u> The first random number for Trial 1 is 9.
Jin's rule gives a time between passengers of 4 minutes.
These values 9 and 4 are ringed in each table.

	Number of minutes between passengers				Number of minutes ferry waits to leave
Trial 1	4	3	3	1	11
Trial 2	2	2	3	3	10
Trial 3	3	4	3	1	11
Trial 4	2	3	4	3	12
Trial 5	3	2	3	4	12
Trial 6	4	3	3	4	14
Trial 7	3	2	1	4	10
Trial 8	3	3	3	3	12
Trial 9					
Trial 10					

[2]

(ii) Use trials 1 to 10 to work out an estimate of the mean time before the ferry leaves when there are already 6 passengers on the ferry.

.... [2]

(iii) Jin estimates that the total time for the first 6 passengers to arrive is 14.5 minutes.

Calculate the mean time for there to be 10 passengers on the ferry.

.... [2]

(b) When the ferry leaves the island it travels 4 km to the mainland.

In this direction of travel

- the speed of the ferry is v km/h
- the speed of the water is 3 km/h
- the speed of travel = speed of the ferry + speed of the water.

(i) When $v = 13$, show that the ferry takes 15 minutes to travel from the island to the mainland.

[1]

(ii) Find a model for the time, t **minutes**, the ferry takes to travel from the island to the mainland. Give your answer in terms of v, in its simplest form.

.... [2]

(iii) Sketch a graph of your model for $0 \leqslant v \leqslant 40$.

[2]

(iv) Find t when the speed of travel is 25 km/h.

.... [2]

5 In this question, there are no passengers on the ferry.

Jin updates his model in **Question 4(a)** in the following way:

- The ferry is at the island.
- The passengers now arrive in groups.
- The ferry travels to the mainland as soon as a group arrives making the total number of passengers 10, 11 or 12.

(a) **(i)** Jin uses the two-digit numbers from 00 to 99 to make a second rule. This rule gives the number of passengers in a group.

Complete the table.

Two-digit numbers	00 to 19	20 to 64	....
Number of passengers in group	1	2	3

[1]

(ii) He also uses his one-digit rule from **Question 4(a)** for the time between each group arriving.

One-digit numbers	0	1, 2	3, 4, 5, 6, 7	8, 9
Number of minutes between groups	1	2	3	4

Jin uses his rules with these tables of random numbers to model the number of passengers in each group and the time between each group arriving.

Numbers used for	Two-digit random numbers						One-digit random numbers					
Trial 1	08	89	97	24	71	12	2	3	4	4	9	1
Trial 2	27	92	18	10	42	60	2	4	1	1	7	6
Trial 3	45	99	94	40	68	29	5	0	8	0	6	2
Trial 4	32	77	66	26	45	03	9	2	3	8	5	4
Trial 5	96	09	55	83	21	72	4	2	1	6	7	4
Trial 6	02	22	80	18	83	54	9	0	3	2	5	8
Trial 7	88	07	43	48	06	11	5	3	1	2	0	7
Trial 8	98	66	14	45	15	73	3	1	7	5	5	2
Trial 9	02	21	00	18	63	70	9	0	5	5	4	8
Trial 10	51	99	30	55	05	48	8	8	9	0	3	5

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Use the last two rows of each random number table to complete the table of trials.
Jin has completed the first eight trials.

In Trial 1, when five groups have arrived there is a total of 12 passengers and so the ferry leaves.
The sixth group of passengers and the sixth number of minutes are both crossed out.

	Number of passengers in group						Number of minutes between groups						Number of minutes ferry waits to leave
Trial 1	1	3	3	2	3	~~1~~	2	3	3	3	4	~~2~~	15
Trial 2	2	3	1	1	2	2	2	3	2	2	3	3	15
Trial 3	2	3	3	2	~~3~~	~~2~~	3	1	4	1	~~3~~	~~2~~	9
Trial 4	2	3	3	2	~~2~~	~~1~~	4	2	3	4	~~3~~	~~3~~	13
Trial 5	3	1	2	3	2	~~3~~	3	2	2	3	3	~~3~~	13
Trial 6	1	2	3	1	3	~~2~~	4	1	3	2	3	~~4~~	13
Trial 7	3	1	2	2	1	1	3	3	2	2	1	3	14
Trial 8	3	3	1	2	1	~~3~~	3	2	3	3	3	~~2~~	14
Trial 9													
Trial 10													

[3]

(iii) Calculate the difference between the mean number of minutes the ferry waits to leave given by the model in **Question 4(a)(iii)** and this updated model.

.... [2]

(iv) **(a)** Give a reason why modelling the arrival of passengers in groups improves the model in **Question 4(a)**.

....

.... [1]

(b) Write down one way that Jin could improve his model in **Question 5(a)**.

....

.... [1]

(b) Jin wants the total time from when the ferry returns to the island to when it next arrives at the mainland to be no more than 20 minutes.

He estimates the speed, v, at which the ferry must travel using the model in **Question 4(b)** and the mean time given by the model in **Question 5(a)**.

Find the value of v, the speed of the ferry from the island to the mainland.

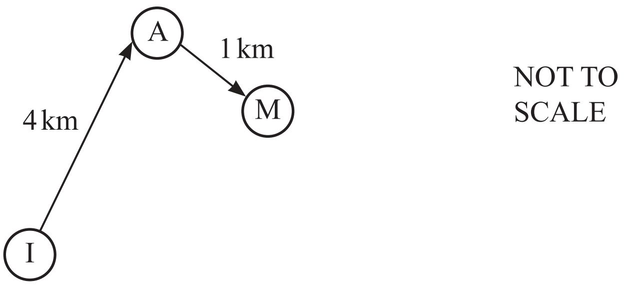
.... [3]

6 Jin changes the route of the ferry so that it makes a stop before it arrives at the mainland.

When there are at least 10 passengers, the ferry

- starts at the island (I)
- travels to another island (A)
- waits 16 minutes then
- travels to the mainland (M).

The diagram shows the ferry journey and the distance, in km, of each part.



The table shows the speed of the ferry and the speed of the water, in km/h, for each part of the journey.

	I to A	A to M
Speed of ferry	v	$v-1$
Speed of water	2	1

(a) Jin finds a model for the total time, t **minutes**, the ferry takes from when it returns to the island to when it next arrives at the mainland.
He uses the mean time from the model in **Question 5(a)**.

His model is $t = \dfrac{k(v^2 + 12v + 4)}{v(v + 2)}$ where k is an integer.

Find the value of k.

.... [4]

(b) For this route, the maximum value of v, the speed of the ferry, is 40 km/h.
Jin wants the total time from when the ferry returns to the island to when it next arrives at the mainland to be no more than 35 minutes.

Work out if it is possible for the ferry to complete the journey in this time.

.... [2]

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