Cambridge Assessment International Education

Cambridge IGCSE™

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9124525720

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/62**

Paper 6 Investigation and Modelling (Extended) **February/March 2024**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 7) and part **B** (Questions 8 to 10).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 7)

MAGIC SQUARES (30 marks)

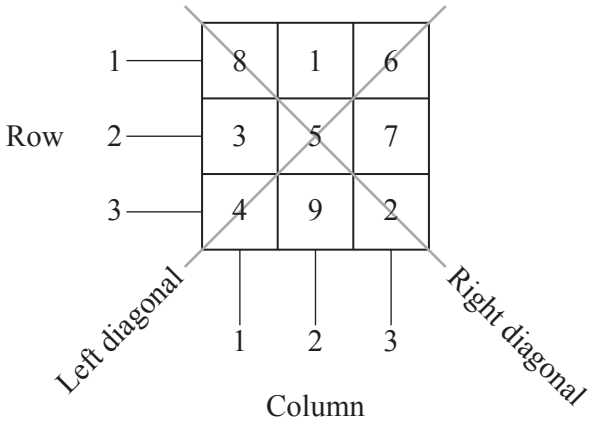
You are advised to spend no more than 50 minutes on this part.

This investigation looks at ways to make a magic square.
A magic square is a grid with a different number in each square.

The numbers in each row, each column or each diagonal all add up to the same total.
This is the *line total*.

Example

This is a 3 by 3 magic square using the integers 1 to 9.
The line total is 15.



Line totals

Row 1	$8+1+6=15$	Column 1	$8+3+4=15$	Left diagonal	$4+5+6=15$
Row 2	$3+5+7=15$	Column 2	$1+5+9=15$	Right diagonal	$8+5+2=15$
Row 3	$4+9+2=15$	Column 3	$6+7+2=15$		

1 A 3 by 3 magic square uses the integers 1 to 9.

Complete the following statements to show that the line total is 15.

The total of the integers 1 to 9 = ...

The total of the integers in all three rows = ...

The line total = ... ÷ ..... = 15

[2]

2 **(a)** This is a different 3 by 3 magic square using the integers 1 to 9.

Complete this magic square.

		4
	5	
	1	

[2]

(b) There are 8 different 3 by 3 magic squares using the integers 1 to 9.
In all these magic squares the integer in the middle square is the same.

(i) What is the connection between the integer in the middle square and the integers 1 to 9?

.....

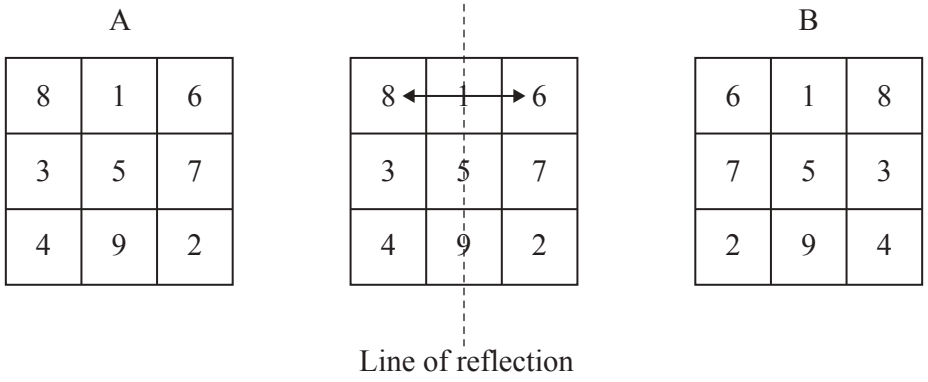
..... [1]

(ii) Explain how to use the integer in the middle square to find the line total.

..... [1]

3 A and B are two magic squares using the integers 1 to 9.
More magic squares can be made using reflection.

B is made by reflecting the position of each integer using a vertical line of reflection.
The line of reflection goes through the middle column of A.
The numbers move from the square in A to the reflected square in B.



(a) Draw a horizontal line of reflection through the middle row of magic square A.

Complete the new magic square C using the horizontal line of reflection.

C

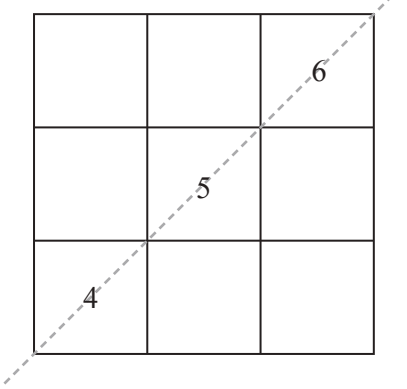
3	5	7

[1]

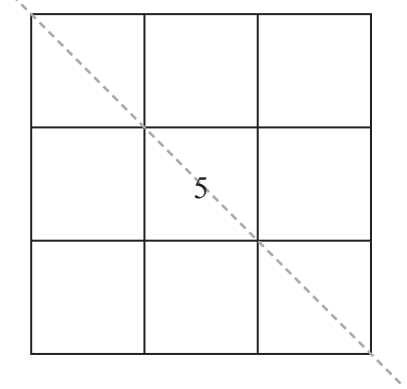
(b) Two different magic squares can be made by reflection in the left and right diagonal lines drawn on A.

Complete these magic squares.

Reflection in left diagonal



Reflection in right diagonal



[2]

4 This is a method to make an n by n magic square where n is odd.

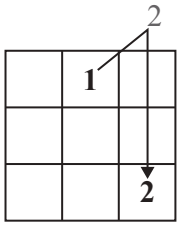
Step 1 Place the smallest integer in the middle square of the top row.

Step 2 Move **up and right** in the direction of the left diagonal to place the next integer.
- **(i)** If you go off the magic square, place the next integer at the opposite end of the row or column you are on.
- **(ii)** If the square that is up and right has an integer in it, go down one square to place the next integer.
- **(iii)** If you are in the top right corner, go down one square to place the next integer.

Step 3 Repeat step 2 until the magic square is complete.

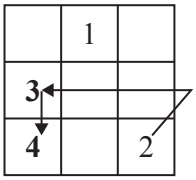
Example using integers 1 to 9

- Put **1** in the middle square of the top row.



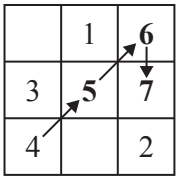
- Move up and right.
  This is off the square, so go to the opposite end of this column and place **2**.

- Move up and right from 2.
  This is off the square, so go to the opposite end of this row and place **3**.



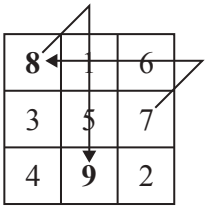
- Move up and right from 3.
  This square has 1 in it, so go down one square from 3 and place **4**.

- Move up and right and place **5** and **6**.



- 6 is in the top right corner so go down one square and place **7**.

- Move up and right from 7.
  This is off the square, so go to the opposite end of this row and place **8**.



- Move up and right from 8.
  This is off the square, so go to the opposite end of this column and place **9**.

(a) Use the method to make a 3 by 3 magic square with the integers 15 to 23.
Step 1 has been done for you.

	15	

[2]

(b) (i) Use the method to complete the 5 by 5 magic square with the integers 1 to 25.
The first six numbers have been placed for you.

		1		
	5			
4	6			
				3
			2	

[2]

(ii) Find the line total for this magic square.

..... [1]

(iii) What is the connection between the line total and the integer in the middle square?

..... [1]

(c) A 7 by 7 magic square uses the integers 1 to 49.

Find the integer in the middle square and the line total.

Integer in the middle square = ...

Line total = ... [2]

5 This is an expression for the line total in an n by n magic square using the integers 1 to n^2.

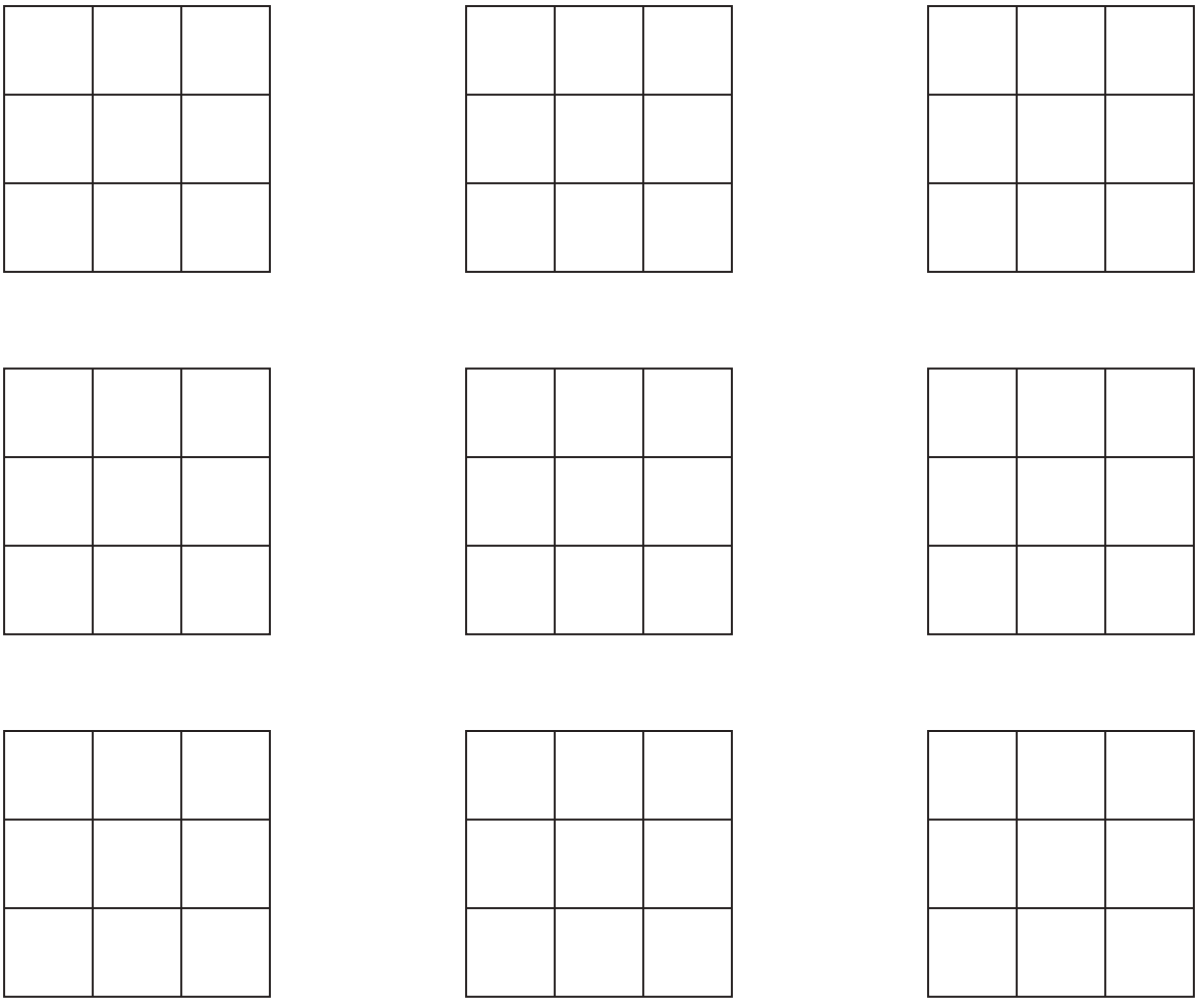
$$\frac{n\left(n^2+1\right)}{2}$$

Find the line total for a magic square using the integers 1 to 81.

..... [2]

6 Different 3 by 3 magic squares are made using the first nine terms of a sequence.
The first term of the sequence is 2.
The nth term of the sequence is $3n-1$.

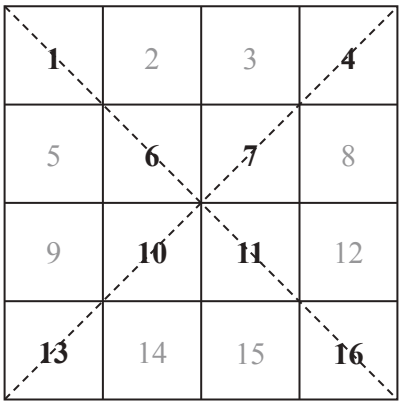
Find six of these magic squares.
Use the method of **Question 4** and reflection to help you.



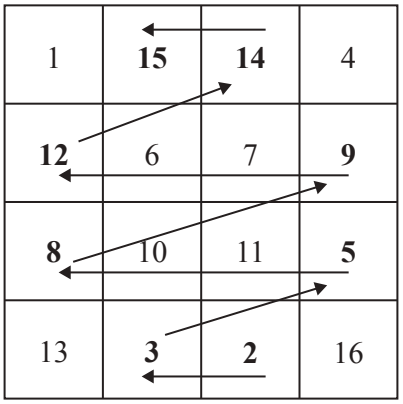
[5]

7 When n is even there is no middle square so we use a different method to make a magic square.

- Draw the two diagonals on the 4 by 4 square.
- Put the smallest integer in the top left square.
- Go from **left to right**, but only write the integers in the squares where there is part of a diagonal.



- Starting with the bottom row write the remaining integers, in order, from **right to left** in the empty squares.



(a) Use this method to complete the 4 by 4 magic square with the consecutive integers k to $k+15$. There is an extra grid for working.

[2]

(b) (i) Find the line total in terms of k.
Give your answer in its simplest form.

..... [2]

(ii) The line total is 254.

Find the value of the largest integer in the third row.

..... [2]

B MODELLING (QUESTIONS 8 to 10)

DELIVERIES BY SCOOTER (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task looks at average speeds of deliveries made by scooter.

A company delivers orders from its warehouses to shops and houses.
All deliveries of small parcels are made by scooter.

8 **(a)** Meera rides a scooter that travels 12 km in 15 minutes on quiet roads.

Show that her average speed on quiet roads is 48 km/h.

[2]

(b) When the roads are busy the time for the 12 km journey is 30 minutes.

Calculate her average speed on busy roads.

..... [2]

(c) Meera makes a delivery.
She travels for 15 minutes on quiet roads and then for 30 minutes on busy roads.

Calculate her average speed for the whole journey.

..... [2]

(d) A warehouse is outside the town.
In her journey from the warehouse Meera always travels the first 4 km on **quiet** roads.
She then travels x km on **busy** roads to make her first delivery.

(i) Show that a model for S, her average speed in km/h, for this whole journey is

$$S = \frac{4+x}{\frac{1}{12}+\frac{x}{24}} .$$

[2]

(ii) Show that the model in **part (i)** simplifies to

$$S = \frac{96+24x}{2+x} .$$

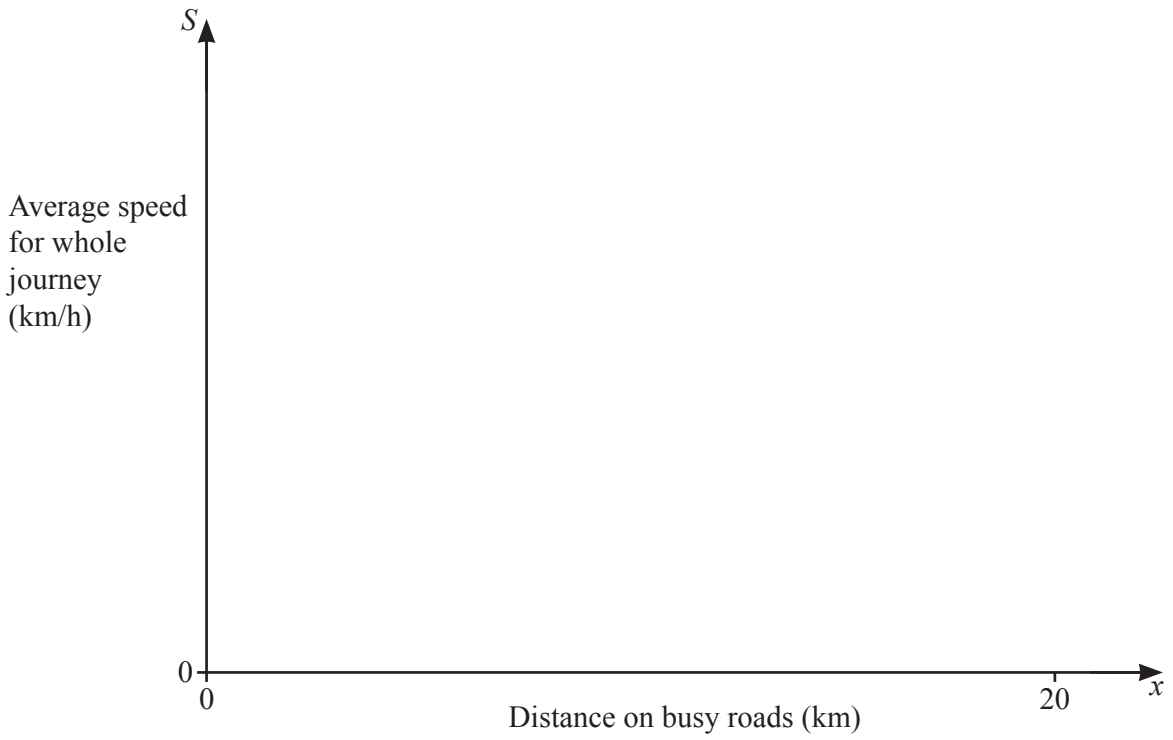
[2]

(e) To make her first delivery Meera leaves the warehouse and travels a total of 10 km on quiet roads and busy roads.

Use the model in **part (d)(ii)** to find Meera's average speed.

..... [3]

(f) Sketch the model for S in **part (d)(ii)** on the axes, for $0 \leqslant x \leqslant 20$.



[2]

(g) Find the total distance of her delivery journey from the warehouse when her average speed is 27 km/h.

..... [2]

9 A second warehouse is in the town.
Meera travels at the same speeds on quiet roads as in **Question 8(a)** and the same speed on busy roads as in **Question 8(b)**.

In her journey from this warehouse Meera always travels the first 3 km on **busy** roads.
She then travels d km on **quiet** roads to make her first delivery.

(a) Find a model for her average speed, Y km/h, when making a delivery from this warehouse.
Do not simplify your model.

..... [2]

(b) Show that your model in **part (a)** simplifies to $Y = \dfrac{144 + 48d}{6 + d}$.

[1]

(c) Find the average speed when $S = Y$ and $x = d$.

..... [2]

10 The model for one delivery from the first warehouse, $S = \frac{96+24x}{2+x}$, is the same as $S = \frac{24(4+x)}{2+x}$.
This model can be changed when there are more deliveries.
Each delivery is a distance of x km on busy roads from the previous delivery.

(a) Rewrite this model for a total of 3 deliveries.

..... [1]

(b) The average time that Meera stops to complete each delivery is 5 minutes.
Meera rewrites the model in **Question 8(d)(i)** to include the times stopped to make all 3 deliveries.

Show that her model is $S = \frac{24(4+3x)}{8+3x}$.

[3]

(c) Meera needs to shorten the average time that she stops so that her average speed for the whole journey is 24 km/h.

Find the total length of time she now stops to make the 3 deliveries.

..... [4]