## Cambridge IGCSE ${ }^{\text {TM }}$



CENTRE NUMBER


CANDIDATE NUMBER


## CAMBRIDGE INTERNATIONAL MATHEMATICS

Paper 6 Investigation and Modelling (Extended)
May/June 2023
1 hour 40 minutes
You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer both part A (Questions 1 to 4 ) and part B (Questions 5 to 6 ).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.


## INFORMATION

- The total mark for this paper is 60 .
- The number of marks for each question or part question is shown in brackets [ ].

The Investigation starts on the next page.

## A INVESTIGATION (QUESTIONS 1 to 4)

## SQUARES IN RECTANGLES <br> (30 marks)

You are advised to spend no more than 50 minutes on this part.
This investigation looks at finding the total number of squares inside a rectangle drawn on a grid.
In this investigation:

- the sides of the rectangles are on the grid lines
- the length of a rectangle is never less than its width.

1 Rectangles of width 1
Length 1
Length 2
Length 3


Complete the statements.
The number of squares in a rectangle of width 1 and length 4 is $\qquad$
The number of squares in a rectangle of width 1 and length $L$ is

2 Rectangles of width 2
Length 2


Length 3


$$
\text { Total }=8 \text { squares }
$$

(a) Draw lines on these rectangles and write the number of squares under each one to show there is a total of 11 squares.

Length 4

squares
of side 1


Total $=11$ squares
(b) (i) Complete the table.

You may use the grid below the table to help you.

| Rectangles of width 2 |  |
| :---: | :---: |
| Length of rectangle | Total number of squares |
| 2 | 5 |
| 3 | 8 |
| 4 | 11 |
| 5 |  |


(ii) Find an expression, in terms of $L$, for the total number of squares in a rectangle of width 2 and length $L$.
(iii) Calculate the total number of squares in a rectangle of width 2 and length 170 .

## 3 Rectangles of width 3

Length 3

(a) Draw lines on these rectangles and write the number of squares under each one to find the total number of squares in a rectangle of width 3 and length 4.

You may not need to use all the rectangles.

Length 4

(b) (i) Complete the table.

You may use the grid to help you.

| Rectangles of width 3 |  |
| :---: | :---: |
| Length of rectangle | Total number of squares |
| 3 | 14 |
| 4 | 26 |
| 5 | 32 |
| 6 |  |
| 7 |  |


(ii) Find an expression, in terms of $L$, for the number of squares in a rectangle of width 3 and length $L$.

4 (a) Complete the table.
Use your answers to Question 1, Question 2(b)(ii) and Question 3(b)(ii) to help you.

| Width of rectangle, $w$ | Expression for total number <br> of squares in terms of $L$ |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 | $10 L-10$ |
| 4 |  |
| 5 | $21 L-35$ |
| 6 |  |

(b) The expressions in the table have two terms.
(i) The first term is $k L$, where $k$ is an integer.

Find an expression for $k$ in terms of $w$.
(ii) The second term in the expression is a constant.

The constant is in the form $a w^{3}+b w$, where $a$ and $b$ are both fractions.
Find the value of $a$ and the value of $b$.
Write down the expression for the constant in terms of $w$.

$$
a=
$$

$$
b=
$$

$\qquad$
(iii) Use your expressions in part (i) and part (ii) to find the total number of squares in a rectangle of width 10 and length 11 .

## B MODELLING (QUESTIONS 5 to 6)

## CARDBOARD BOXES (30 marks)

You are advised to spend no more than 50 minutes on this part.
This task looks at the dimensions of storage boxes.
A company stores metal bolts in closed boxes which are square-based cuboids.
The boxes are made of 5 mm thick cardboard.
The external side of the base is $L \mathrm{~cm}$ and the external height is $H \mathrm{~cm}$.

The volume of a box is all the space that the box takes up. The capacity of a box is all the space inside it.


5 (a) Show that the internal dimensions of the box are $(L-1) \mathrm{cm}$ and $(H-1) \mathrm{cm}$.
(b) Write down a formula for $C$, the capacity of the box, in terms of $L$ and $H$.
(c) The bolts are so heavy that an extra piece of cardboard of thickness 5 mm is placed in the bottom of the box to increase its strength.

Change your model for $C$ in part (b) to include this extra piece of cardboard.
(d) The external area of the base of the box is $900 \mathrm{~cm}^{2}$.

The height of the box is 5 cm longer than the length of the base.
(i) Use your model in part (c) to calculate the capacity of the box. Write down all the figures on your calculator.
(ii) Calculate the difference between the capacity and the volume of the box.

6 The net of a 3D shape is what it looks like when it is opened out flat. This is a net for a box with the extra piece of cardboard.

(a) Show that the formula for $A$, the area of cardboard including the extra piece, is

$$
A=3 L^{2}+4 L H-2 L+1 .
$$

You may use the diagram to help you.
(b) The box with external dimensions $L \mathrm{~cm}, L \mathrm{~cm}$ and $H \mathrm{~cm}$ has a volume of $12500 \mathrm{~cm}^{3}$.
(i) Write an expression for $H$ in terms of $L$.
(ii) Show that the model for $A$ in part (a) in terms of $L$ is

$$
A=3 L^{2}+\frac{50000}{L}-2 L+1 .
$$

(iii) Sketch the model in part (ii) on the axes, for $0<L \leqslant 60$.

(iv) Box A has the minimum area of cardboard.

Write down the length of the base of the box and the area of the cardboard.

$$
\begin{align*}
\text { Length } & = \\
\text { Area } & = \tag{2}
\end{align*}
$$

(v) Box B is a cube with volume $12500 \mathrm{~cm}^{3}$.

Find the area of the cardboard when the box is a cube.
(vi) Find which box, A or B, has the greater height. Calculate the difference in height.
(vii) Find which box, A or B , has the greater capacity. Calculate the difference in capacity.

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