## Cambridge IGCSE ${ }^{\text {TM }}$



## CAMBRIDGE INTERNATIONAL MATHEMATICS

Paper 4 (Extended)
May/June 2023
2 hours 15 minutes
You must answer on the question paper.
You will need: Geometrical instruments

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly and you will be given marks for correct methods, including sketches, even if your answer is incorrect.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- For $\pi$, use your calculator value.


## INFORMATION

- The total mark for this paper is 120 .
- The number of marks for each question or part question is shown in brackets [ ].


## Formula List

For the equation

$$
a x^{2}+b x+c=0 \quad x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

Curved surface area, $A$, of cylinder of radius $r$, height $h$.
$A=2 \pi r h$

Curved surface area, $A$, of cone of radius $r$, sloping edge $l$.
$A=\pi r l$

Curved surface area, $A$, of sphere of radius $r$.
$A=4 \pi r^{2}$

Volume, $V$, of pyramid, base area $A$, height $h$.
$V=\frac{1}{3} A h$

Volume, $V$, of cylinder of radius $r$, height $h$.
$V=\pi r^{2} h$

Volume, $V$, of cone of radius $r$, height $h$.

Volume, $V$, of sphere of radius $r$.
$V=\frac{1}{3} \pi r^{2} h$
$V=\frac{4}{3} \pi r^{3}$


$$
\begin{aligned}
& \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
& a^{2}=b^{2}+c^{2}-2 b c \cos A \\
& \text { Area }=\frac{1}{2} b c \sin A
\end{aligned}
$$

## Answer all the questions

1

(a) Translate triangle $A$ by $\binom{-6}{4}$. Label the image $C$.
(b) Describe fully the single transformation that maps triangle $A$ onto triangle $B$.
$\qquad$
$\qquad$
(c) Reflect triangle $A$ in the line $x=-2$. Label the image $D$.
(d) Enlarge triangle $A$ by scale factor -1 , centre ( 0,0 ). Label the image $E$.

2 A triathlon race consists of three parts:

- a 1500 m swim
- a 40 km bike ride
- a 10 km run.
(a) John swims the 1500 m in 25 minutes.

Find his average speed, in $\mathrm{km} / \mathrm{h}$, for this swim.
(b) John completes the 40 km bike ride at an average speed of $32 \mathrm{~km} / \mathrm{h}$.

Find the time, in minutes, for John to complete this bike ride.
$\qquad$ $\min [2]$
(c) John completes the whole race at an average speed of $20.6 \mathrm{~km} / \mathrm{h}$.

Find the average speed, in $\mathrm{km} / \mathrm{h}$, for John to complete his 10 km run.

3 The table shows the marks of 12 students in a French examination and a Spanish examination.

| French mark (x) | 17 | 23 | 28 | 32 | 37 | 42 | 57 | 61 | 77 | 82 | 94 | 96 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Spanish mark (y) | 26 | 22 | 33 | 46 | 41 | 53 | 62 | 67 | 66 | 75 | 83 | 95 |

(a) Find the median Spanish mark.
(b) Find the mean French mark.
$\qquad$
(c) Find the equation of the regression line for $y$ in terms of $x$.

$$
y=
$$

(d) Use your equation to estimate the Spanish mark when
(i) the French mark is 50
$\qquad$
(ii) the French mark is 6 .
$\qquad$
(e) Which French mark, 50 or 6, is likely to give the most reliable Spanish mark? Give a reason for your answer.
$\qquad$ because $\qquad$

4 (a) $\$ x$ is divided in the ratio $3: 5$. The larger share is $\$ 42$.

Find the value of $x$.

$$
\begin{equation*}
x= \tag{2}
\end{equation*}
$$

(b) (i) Increase 124 by $16 \%$.
$\qquad$
(ii) The price of a coat is reduced by $\frac{2}{9}$ in a sale.

The new price of the coat is $\$ 73.50$.
Find the original price of the coat.

$$
\$
$$

(c) Xiong invests $\$ 2000$ in Bank $A$ which pays simple interest at a rate of $3 \%$ each year.

Find the total amount of interest Xiong receives at the end of 5 years.

$$
\$
$$

(d) Wendi invests $\$ 400$ in Bank $B$ which pays compound interest at a rate of $1.6 \%$ each year. Find the total amount of interest Wendi receives at the end of 3 years.
(e) Pedro invests $\$ 1000$ in Bank $C$ for 18 years.

Pedro also invests $\$ 1000$ in Bank $D$ for 18 years.

Bank $C$ pays simple interest at a rate of $x \%$ each year.
Bank $D$ pays compound interest at a rate of $0.7 x \%$ each year.
At the end of 18 years Pedro has exactly the same amount of money in Bank $C$ and Bank $D$.
(i) Show that $1+\frac{18 x}{100}=\left(1+\frac{0.7 x}{100}\right)^{18}$.
(ii) Given that $5<x<7$, use a graphical method to find $x$.

$x=$


In the diagram, $A B$ is parallel to $E D$.
$A C D$ and $B C E$ are straight lines.
$A B=50 \mathrm{~cm}, B C=55 \mathrm{~cm}$ and $A C=64 \mathrm{~cm}$.
(a) Show that angle $A C B=49.0^{\circ}$ correct to one decimal place.
(b) Use the sine rule to calculate angle $C A B$.

## Angle $C A B=$

(c) Calculate the area of triangle $A B C$.
(d) $A C=\frac{2}{3} A D$

Calculate the area of triangle $C D E$.

6 (a) Solve.

$$
7 x-5=3 x+13
$$

$$
\begin{equation*}
x= \tag{2}
\end{equation*}
$$

(b) Solve.

$$
4(2 x-3)=3(1-2 x)
$$

$$
\begin{equation*}
x= \tag{3}
\end{equation*}
$$

(c) Solve.

$$
\frac{3 x+2}{8}=\frac{2}{3 x+2}
$$

$$
x=. . . . . . . . . . . . . . . . . ~ o r ~ x=.
$$

(d) Solve.

$$
1-2 x^{2}=5 x-1
$$

Give your answer correct to two decimal places.
$\qquad$ or $x=$
(e) $\quad \log x=1+4 \log y$

Find $x$ in terms of $y$.

$$
\begin{equation*}
x= \tag{3}
\end{equation*}
$$

(f) There are 12 balls in a bag, $n$ of them are blue.

A ball is taken from the bag at random and replaced.
The probability that the ball is blue is $p$.
6 more blue balls are added to the bag.
A ball is taken from the bag at random.
The probability that this ball is blue is $2 p$.
Find the value of $p$.

$$
p=
$$


(a) On the diagram, sketch the graph of $y=\mathrm{f}(x)$.
(b) Write down the $x$-coordinates of the points where the curve meets the $x$-axis.

$$
x=
$$

$\qquad$ or $x=$
(c) Write down the coordinates of the local maximum point.
$\qquad$
(d) The line $y=0.005 x$ intersects the curve $y=\left|2 \cos (x-45)^{\circ}\right|-1$ three times.
(i) Find the $x$-coordinates of the points of intersection.

$$
\begin{equation*}
x= \tag{3}
\end{equation*}
$$

$\qquad$ or $x=$ $\qquad$ or $x=$ $\qquad$
(ii) Solve the inequality.

$$
\left|2 \cos (x-45)^{\circ}\right|-1>0.005 x
$$

8 (a) Solve the simultaneous equations.

$$
\begin{aligned}
& 5 x-4 y=13 \\
& 3 x+2 y=-1
\end{aligned}
$$

You must show all your working.

$$
\begin{align*}
& x=\text {................................................ } \\
& y=. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~
\end{align*}
$$

(b) $\quad \mathrm{f}(x)=3 x+1 \quad \mathrm{~g}(x)=\frac{1}{2 x-3}, x \neq 1.5$
(i) Find $\mathrm{f}(-2)$.
$\qquad$
(ii) Find $\mathrm{f}(\mathrm{f}(x))$, giving your answer in its simplest form.
$\qquad$
(iii) Solve $\mathrm{g}(\mathrm{f}(x))=\frac{1}{5}$.

$$
x=
$$

9 There are 80 students in a school year, 44 boys and 36 girls.
Each student chooses their favourite sport.
The number of boys and the number of girls choosing each sport is shown in the table.

|  | Athletics | Football | Hockey | Swimming |
| :--- | :---: | :---: | :---: | :---: |
| Boys | 12 | 16 | 8 | 8 |
| Girls | 5 | 3 | 17 | 11 |

(a) A student is chosen at random from the 80 students.

Find the probability that the student chosen is
(i) a girl whose favourite sport is athletics
(ii) a boy whose favourite sport is not football.
$\qquad$
(b) One of the girls is chosen at random.

Find the probability that her favourite sport is hockey.
(c) Three of the boys are chosen at random.
(i) Find the probability that one of the boys chooses athletics, one of them chooses football and the other chooses swimming.
(ii) Calculate the probability that the three boys each have a different favourite sport.

$A, B, C$ and $D$ lie on a circle, centre $O$.
$D E$ is a tangent to the circle at $D$.
$A C E$ is a straight line.
Find
(a) angle $A O C$

$$
\begin{equation*}
\text { Angle } A O C= \tag{1}
\end{equation*}
$$

(b) angle $O A C$

Angle $O A C=$
(c) angle $A D C$

Angle $A D C=$
(d) angle CAD.

11


NOT TO
SCALE

The diagram shows a cuboid with base $A B C D$.
$A B=20 \mathrm{~cm}, B C=34 \mathrm{~cm}$ and $C E=16 \mathrm{~cm}$.
Water is poured into the cuboid to a height of 8 cm .
(a) Find the volume of water in the cuboid.
$\qquad$
$\mathrm{cm}^{3}$
(b) A sphere of radius 4 cm is placed so that it rests on the base of the cuboid. The water level is now $q \mathrm{~cm}$ above the base of the cuboid.

Find the value of $q$.
(c) The sphere is removed from the cuboid.

15 identical cubes of side $x \mathrm{~cm}$ are placed so that they rest on the base of the cuboid.
(i) Find the maximum value of $x$.

$$
x=
$$

(ii) The water level is now $p \mathrm{~cm}$ above the base of the cuboid.

Find the maximum value of $p$.

$$
p=
$$

12


The diagram shows two right-angled triangles $A B C$ and $C B D$.
$A B=B C, A C=\sqrt{200} \mathrm{~cm}$ and angle $B D C=30^{\circ}$.
Find the perimeter of triangle $A C D$.

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